The **Erlangen National High Performance Computing Center** (NHR@FAU) is looking for a

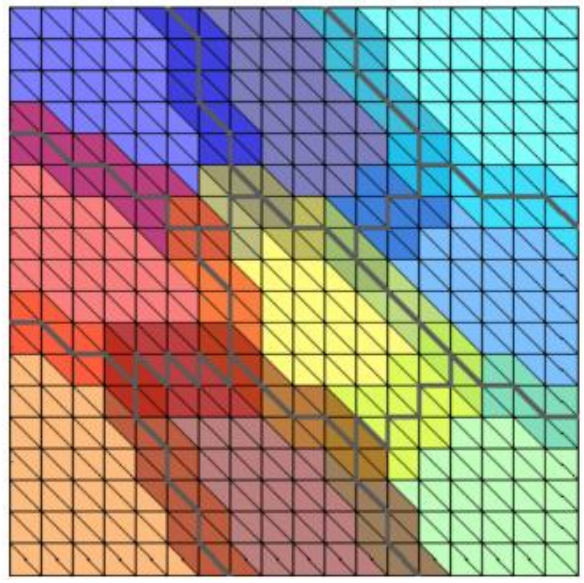
Master thesis student for

Efficient Inexact Subdomain Solvers for Domain Decomposition Methods with RACE

The thesis will be supervised collaboratively by the Research division at NHR@FAU and the Numerical Analysis group at TU Delft.

Description

Partial differential equations (PDEs) are widely used in scientific and engineering problems, where discretization leads to large, sparse linear systems $Ax = b$. As system size increases, direct solvers become impractical due to high memory and computation costs, so iterative methods such as Krylov subspace solvers are employed [1]. However, these methods can converge slowly for large systems, motivating the use of (domain decomposition-based) preconditioners $M^{-1} \approx A^{-1}$.



Let $\{\Omega_i\}, i = 1, \dots, N$ be a decomposition of our domain Ω into overlapping subregions (see image). This project focuses on overlapping Schwarz preconditioners of the form

$$M^{-1} = \sum_{i=1}^{N} R_i^T A_i^{-1} R_i,$$

where R_i restricts the i-th overlapping subdomain and $A_i = R_i A R_i^T$. Using A_i^{-1} in the preconditioner (i.e., an "*exact*" solver) quickly becomes the performance bottleneck. This thesis explores "*inexact*" solvers as a cost-effective alternative, and aims to evaluate the performance vs. accuracy tradeoff of different inexact solvers. We will use the FROSch package [2] within the Trilinos framework as the domain decomposition preconditioner. New hardware-aware algorithms, such as cache-blocking matrix-power kernels provided by the Recursive Algebraic Coloring Engine (RACE), will be taken into account [3].

[1] Y. Saad. *Iterative Methods for Sparse Linear Systems*. doi: 10.1137/1.9780898718003.

[2] A. Heinlein, et al. *FROSch: a fast and robust overlapping Schwarz domain decomposition preconditioner based on Xpetra in Trilinos*. doi: 10.1007/978-3-030-56750-7_19.

[3] C. L. Alappat, et al. *Level-based Blocking for Sparse Matrices: Sparse Matrix-Power-Vector Multiplication*. doi: 10.1109/TPDS.2022.3223512.

Within the master thesis, the focus will be in the following areas:

- Literature review on domain decomposition methods and inexact subdomain solvers.
- Installation of FROSch on DelftBlue (TU Delft) and/or Fritz (NHR@FAU) supercomputers.
- Interfacing FROSch with the RACE framework.
- Systematic performance evaluation of the exact vs. inexact subdomain solvers on DelftBlue and Fritz.

Working on these topics, you will have the chance to work with **state-of-the-art supercomputing technology** in a scientific environment. We foster **scientific thinking** and proper **data presentation skills**.

Required skills

- Profound knowledge of C++
- Experience using numerical libraries/frameworks (e.g., Intel MKL, BLAS, Eigen…)
- Knowledge of high-performance parallel computer architecture and code parallelization with OpenMP (MPI is a plus)
- Knowledge of basic performance modeling strategies (e.g., Roofline)
- Excellent communication skills in English

Please direct any inquiries or applications to

Dane Lacey <dane.c.lacey@fau.de>
Georg Hager <georg.hager@fau.de>
Gerhard Wellein <gerhard.wellein@fau.de>