Algebraic Programming

for High Performance Auto-Parallelised Solvers

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Humble and Hero Programming

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- **Humble** programmers: achieve maximum productivity.

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  - increasingly complex: many-core, heterogeinity, **deeper NUMA** effects, memory walls, and **low memory capacity** per core.

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Increasingly many hardware targets,

increasingly heterogeneous hardware:

a software productivity crisis is looming

Historical context

- The Design and Analysis of Computer Algorithms, Aho, Hopcroft, Ullman (1974)
- Introduction to Algorithms (first edition only), Cormen, Leiserson, Rivest (1990)
- **Elements of Programming**, Alexander Stepanov & Paul McJones (2009)
- **Graph Algorithms in the Language of Linear Algebra**, Jeremy Kepner & John Gilbert (2011)
- **From Mathematics to Generic Programming**, Alexander Stepanov & Daniel Rose (2015)
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- A C++ GraphBLAS, Y., Suijlen, Di Nardo, Nash (2017)
- **Algebraic Programming** (2021 onwards)
- ...

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- automatically optimise code based on algebraic information;
- allow only scalable expressions.

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Three ALP concepts: algebraic *containers*, *structures*, and *primitives*.

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Containers are similar to the standard template library (STL):

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grb::Vector< double > x( n ), y( n ), z( n );
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Elements may be **any POD type**, containers have **capacities**

```
grb::Vector< std::pair< int, double > > pairs( n );
grb::Vector< bool > s( n, 1 );        // nz cap: one
grb::Matrix< void > L( n, n, nz );  // nz cap: nz
```

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Elements may be **any POD type**, containers have **capacities** and **IDs**:

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grb::Vector< std::pair< int, double > > pairs( n );
grb::Vector< bool > s( n, 1 );       // nz cap: one
grb::Matrix< void > L( n, n, nz ); // nz cap: nz
std::cout << "s has ID " << grb::getID( s ) << "\n";
```

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More complex structures may be **composed**:

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grb::Monoid<
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grb::Monoid<
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grb::Semiring<
   grb::operators::add< double >,
   grb::operators::mul< double >,
   grb::identities::zero, grb::identities::one
> mySemiring;
```

Basics

Algebraic primitives operate on algebraic containers:

- `grb::set( x, 1.0 );` // $x_i = 1, \ \forall i$
- `grb::setElement( y, 3.0, n/2 );` // $y_{n/2} = 3$

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Semantics may change based on **required** algebraic structures:

- grb :: eWiseApply(z, x, x, minOp); // $z_i = \min\{x_i, x_i\}, \ \forall i$
- grb :: eWiseApply(z, x, y, minOp); // $z_{n/2} = \min\{x_{n/2}, y_{n/2}\}$

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- grb :: mxv(y, A, x, mySemiring); // y += Ax; in-place

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- grb::mxv(y, A, x, mySemiring); // $y \mathrel{+}= Ax$; in-place

All primitives except 'getters' such as grb::{ size ,nrows,nnz,capacity }:

- allow input & output **masks**, **descriptors**, and **phase** arguments;

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All primitives except 'getters' such as grb :: { size , nrows, nnz, capacity }:

- allow input & output **masks**, **descriptors**, and **phase** arguments;
- return **error codes** such as for mismatching dimensions.

Example: PageRank

Inner PageRank loop in ALP, **following a textbook** definition:

```
beta = gamma = 0;
foldl< invert_mask >( gamma, pr, r, add );          // gamma = pr( !r ) * e^T
eWiseApply( u, pr, r, mul );                         // u = pr .* r
gamma = ( alpha * gamma + 1 - alpha ) / n;           // standard scalar arith.
set( t, 0 ); vxm( t, u, L, plusTimes );              // t = u * L
foldl( t, gamma, add );                              // t = t .+ gamma
dot( beta, pr, t, add, absDiff );                    // beta = || pr - t ||_1
std::swap( pr, t ); ++iter;
if( beta <= tol || iter == max_iter ) { break; }
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dot( beta, x, y, add, operators::conj_right_mul< T >() );
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Selecting the **sequential** auto-vectorising backend:

```
grbcxx -o myProgram myProgram.cpp
grbrun ./myProgram datasets/west0497.mtx
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Selecting the **shared-memory parallel** **auto-vectorising** backend:

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grbcxx --backend reference_omp -o myProgram myProgram.cpp
grbrun -b reference_omp ./myProgram datasets/west0497.mtx
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Selecting the **1D distributed-memory parallel** backend (4 nodes):

```
grbcxx -b bsp1d -o myProgram myProgram.cpp
grbrun -b bsp1d -np 4 ./myProgram datasets/west0497.mtx
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Selecting the **hybrid shared and dist. parallel** backend (10 nodes):

```
grbcxx -b hybrid -o myProgram myProgram.cpp
grbrun -b hybrid -np 10 ./myProgram datasets/west0497.mtx
```

The nonblocking backend

Suppose we compute $s = r + \alpha v$ over a given `semiring`:

1) `grb::set( s, r );` // $s = r$
2) `grb::eWiseMul( s, alpha, v, semiring );` // s += $alpha$.* v

Blocking execution: the vector s is accessed *twice*

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1) grb :: set(s, r); // $s = r$
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Manual fusion (Y. et al., '20): performance ✓

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grb::eWiseLambda( [ &s, &r, &alpha, &v, &ring ] (const size_t i) {
    grb::apply( s[ i ], alpha, v[ i ], ring.getMultiplicativeOperator() );
    grb::foldl( s[ i ], r[ i ], ring.getAdditiveOperator() );
}, s, r, v );
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Ref.: A C++ GraphBLAS: specification, implementation, parallelisation, and evaluation by Y., D. Di Nardo, J. M. Nash, and W. J. Suijlen (2020).

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Automatic non-blocking mode (Mastoras et al., '22):

Ref.: Nonblocking execution in GraphBLAS by Aristeidis Mastoras, Sotiris Anagnostidis, and Y. in 2022 IEEE International Parallel and Distributed Processing Symposium Workshops (IPDPSW).
Ref.: —, "Design and implementation for nonblocking execution in GraphBLAS: tradeoffs and performance", ACM TACO, 2023.

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Blocking execution: the vector s is accessed *twice*; performance ✗

Manual fusion (Y. et al., '20): performance ✓, not very humble ✗

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grb::eWiseLambda( [ &s, &r, &alpha, &v, &ring ] (const size_t i) {
    grb::apply( s[ i ], alpha, v[ i ], ring.getMultiplicativeOperator() );
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}, s, r, v );
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Automatic non-blocking mode (Mastoras et al., '22):

- *lazily* evaluate ALP/GraphBLAS calls, no ALP program changes!

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Suppose we compute $s = r + \alpha v$ over a given `semiring`:

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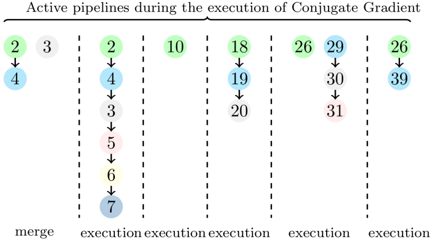


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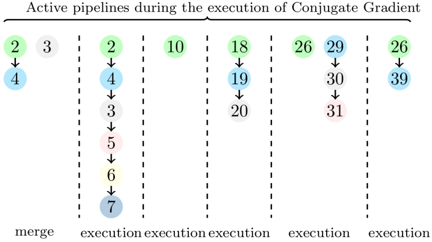

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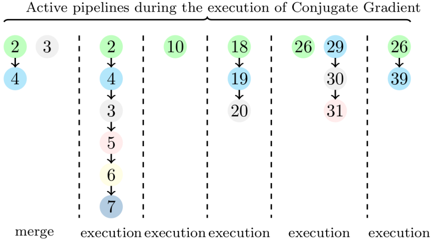

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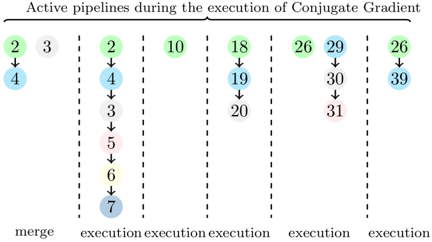


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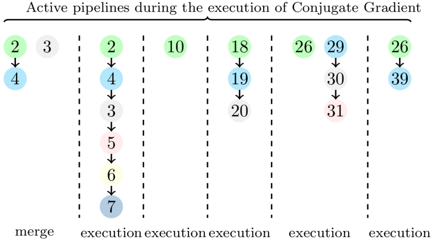


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Performance

For the nonblocking backend, expect:

- speedup *up to* pipeline depth if $nz = \Theta(n)$;

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Similar results for PageRank and sparse deep neural network inference.

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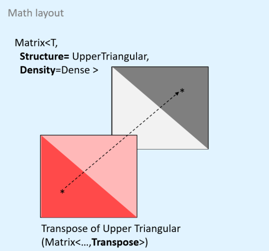

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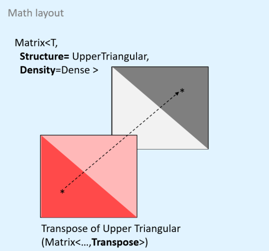

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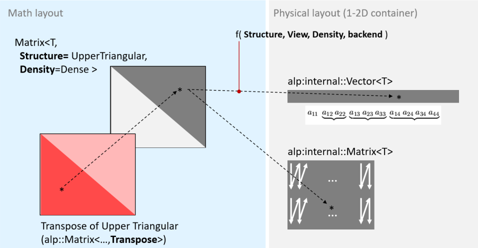

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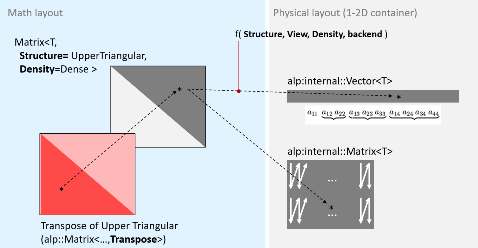

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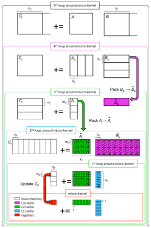

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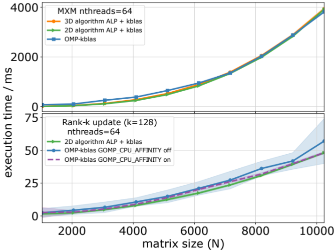
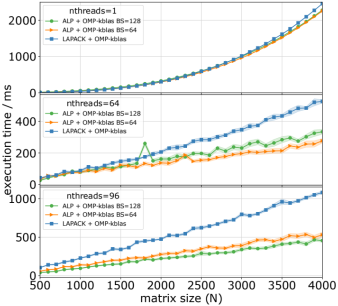
Dense matrix–matrix multiplication (left) and Cholesky decomposition (right)





ALP/Dense for solvers

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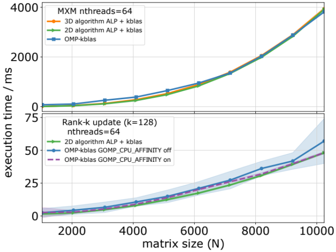

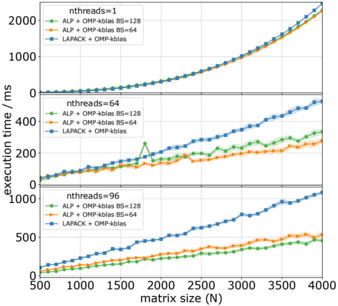

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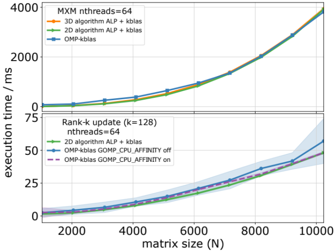

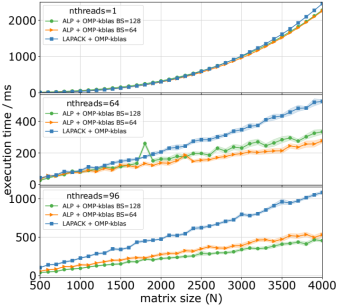

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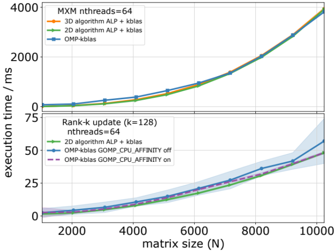

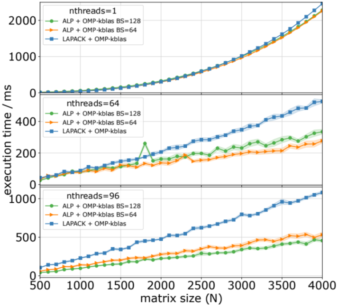

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- for sparse LA, hard-coded fusion in CG usually only **minor impact**.

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      grb::Vector< IOType > &buffer3,  grb::Vector< IOType > &buffer4
);
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- CRS-compatible sparse iterative solvers:

```
double *x, *b, *a_vals; int *a_cols, *a_offs;
// ...

sparse_cg_handle_t handle;
sparse_cg_init( &handle, n, a_vals, a_cols, a_offs );

sparse_cg_solve( handle, x, b );
sparse_cg_destroy( handle );
```

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- CRS-compatible solvers with user-defined preconditioning:

```
double *x, *b, *a_vals; int *a_cols, *a_offs;
int my_preconditioner( double * out, const double * in, void * data );
// ...
sparse_cg_handle_t handle;
sparse_cg_init( &handle, n, a_vals, a_cols, a_offs );
sparse_cg_set_preconditioner( handle, my_preconditioner, data );
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```

CG: transition and embrace results, v0.8 (prelim.)

Transition vs. embrace path performance on 2-socket ARM, 96 cores:

- non-preconditioned CG, 1000 iterations, not converged;
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- in order: embrace

ms./iter	ecology2	apache2	G3circuit	Emilia923	Serena	Queen4147
Eigen	15.1	10.5	22.3	20.0	28.1	99.2
ALP blk.	1.67	1.53	2.45	5.93	8.65	36.4
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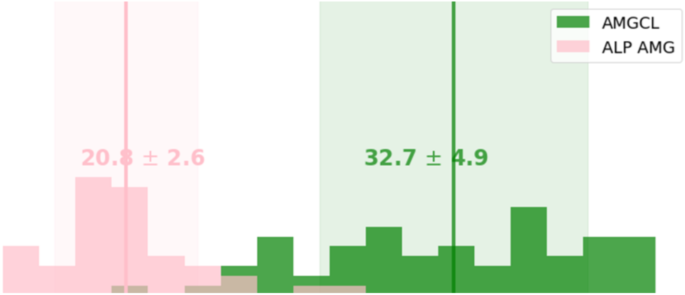
ALP up to $28.5\times$ faster vs. Eigen and 6.96, $2.01\times$ vs. hand-optimised.

ALP PCG with AMG preconditioning

AMG PCG in ALP vs. AMGCL on an internal problem:

- Matrix size n approx. $217M$, 460M nonzeroes.
- explicit coarsening / restriction matrices applied within ALP;
- compiled using the non-blocking backend.



(obtained using the 2-socket ARM machine, 96 cores; similar for x86)

ALP PCG with ILU(tresh)

Tresholded ILU PCG in ALP vs. Eigen on the same internal problem:

- forward/backward solves $Lx = b$ and $L^T y = x$ are outside of ALP;
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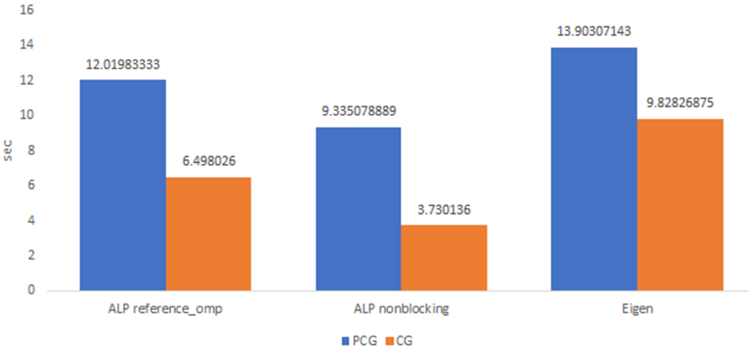

(obtained on a 2-socket Intel x86 machine, 32 threads)

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The ALPs:

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One software stack, many humble interfaces, hero performance

Ref.: Y., "Humble Heroes", Communications of Huawei Research.

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Publications:

- Suijlen, Y.: Lightweight Parallel Foundations: a model-compliant communication layer (2019);
- Y., Di Nardo, Nash, Suijlen: A C++ GraphBLAS: specification, implementation, parallelisation, and evaluation (2020);
- Mastoras, Anagnostidis, Y.: Nonblocking execution in GraphBLAS, IPDPSW (2022);
- Chelini, Barthels, Bientinesi, Copic, Grosser, Spampinato: MOM: Matrix Operations in MLIR, HiPEAC IMPACT workshop (2022);
- Y.: Humble Heroes, Communications of Huawei Research (2023, to appear);
- Mastoras, Anagnostidis, Y.: Design and implementation for nonblocking execution in GraphBLAS: tradeoffs and performance, ACM TACO (2023);
- Scolari, Y.: Effective implementation of the High Performance Conjugate Gradient benchmark on ALP/GraphBLAS, IPDPSW (GrAPL 2023);
- Spampinato, Jelovina, Zhuang, Y.: Towards Structured Algebraic Programming, ACM ARRAY (2023);
- Papp, Anegg, Y.: Partitioning Hypergraphs is Hard: Models, Inapproximability, and Applications, ACM SPAA (2023);
- Papp, Anegg, Y.: DAG scheduling in the BSP model (submitted, 2023);
- Pasadakis, et al., Nonlinear spectral clustering with C++ GraphBLAS, extended abstract, IEEE HPEC (2023, **outstanding short paper**);
- Papp, Anegg, Karanasiou, Y.: Efficient Multi-Processor Scheduling in Increasingly Realistic Models (submitted, 2023).

Backup slides

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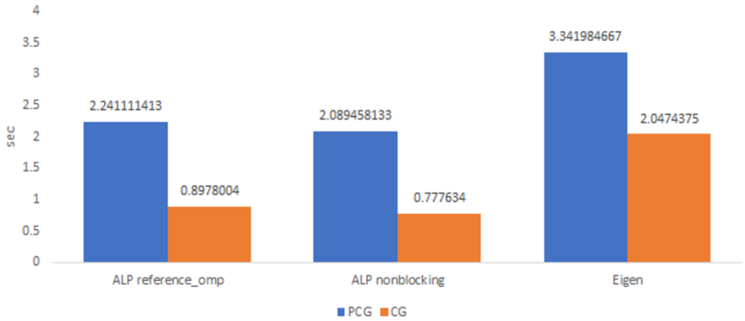
ILU(tresh) PCG on x86

Two-socket Intel x86, smaller problem

- Matrix size *n* approx. 44M, 100M nonzeroes.



(obtained on a 2-socket Intel x86 machine, 32 threads)

Here, PCG Eigen: 3.36, Eigen+Hdagg: 2.08, ALP+Hdagg: 0.778 s.

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ALP generates the following libs:

- sparseblas and sparseblas_shmem_parallel
- alp_cspblas and alp_cspblas_shmem_parallel
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// ...ALP computations...
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- ALP backend implementations and algorithms remain unchanged;
- via auto-vectorising serial and nonblocking parallel backends.

Key enabler: a **native interface**

```
double *x_raw, *a; int *ja, *ia; size_t m, n;

grb::Vector< double > x = wrapRawVector< double >( m, x_raw );
grb::Matrix< double > A = wrapCRSMatrix( a, ja, ia, m, n );

// ...ALP computations...

grb::wait( A, x );
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Libraries generated by ALP

ALP generates the following libs:

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- alp_cspblas and alp_cspblas_shmem_parallel
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HPCG benchmark, dual-socket ARM, 96 cores, maximum problem size

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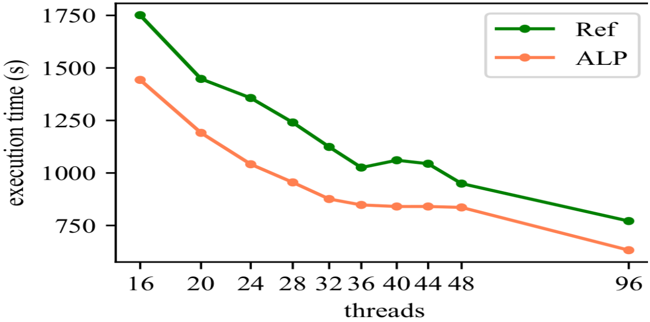
Comparison, using the blocking ALP backend:



Ref. Scolari, Y.: "Effective implementation of the High Performance Conjugate Gradient benchmark on ALP/GraphBLAS", GrAPL at IPDPSW (to appear, 2023)

Performance

Scale-out performance of graph algorithms, using the hybrid backend:

- Clueweb12 link matrix, approx. 978M vertices and 42.5B edges

	Ivy Bridge nodes						
	4	5	6	7	8	9	10
Input	1524	1271	1067	943	691	662	537
4-hop reachability BFS	48.8	110	54.8	99.6	83.0	74.2	23.3
20-hop reachability BFS	404	280	231	323	221	230	160
PageRank	13.3	10.3	9.68	8.00	21.0	22.9	21.6

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Transition path: CG, x86, ALP v0.5

ms. (and speedup) per CG iteration on x86 (2 CPUs, 88 hyperthreads):

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Eigen	0.139	1.00×	77.0	0.79×
GSL	0.864	0.16×	767	0.08×
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- up to 1.5× faster than SuiteSparse:GraphBLAS.

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- guide programmers to express the best possible algorithm;
- gauge scalability: compute resources vs. problem size;
- expose trade-off opportunities: e.g., speed vs. memory;
- automatic choice of algorithms and backends.

Performance semantics

Every ALP program can be **systematically costed**:

Primitive	Work	Ops	Data movement	Reductions
setElement(x, y, i)	1	-	1	no
set(x, y)	$\min\{n, nz_x + nz_y\}$	-	$nz_x + nz_y$ or $n + nz_y$	no
clear(x)	nz_x	-	nz_x	no
apply$(z, x, y, \odot/M)$	$\min\{n, nz_x + nz_y\}$	$nz_{x\cap y}$	$2\min\{n, nz_x + nz_y\} + nz_{x\cup y}$	no
foldl$(y, x, \odot/M)$ foldr$(x, y, \odot/M)$	nz_x	$nz_{x\cap y}$	$2nz_x$	no
foldl$(y, \alpha, \odot/M)$ foldr$(\alpha, y, \odot/M)$	nz_y	nz_y	nz_y	no
foldl(α, y, M) foldr(y, α, M)	nz_y	nz_y	nz_y	yes
mul(z, x, y, R)	$\min\{nz_x, nz_y\}$	$nz_{x\cap y}$	$2\min\{nz_x, nz_y\} + nz_{x\cap y}$	no
dot$(z, x, y, (M, \odot))$	n	$2n$	$2n$	yes
dot(z, x, y, R)	$\min\{nz_x, nz_y\}$	$2 \cdot nz_{x\cap y}$	$2\min\{nz_x, nz_y\}$	yes

Level-1 primitives and their costs, excluding masking. Similar tables exist for level-2 and level-3 primitives.

Ref.: A C++ GraphBLAS: specification, implementation, parallelisation, and evaluation by Y., D. Di Nardo, J. M. Nash, and W. J. Suijlen (2020).

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Every container has **memory use semantics**:

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grb::Vector< bool > s( n, 1 );
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Capacities:

- are **lower bounds**; grb::capacity(s) ≥ 1;
- may **increase** through grb::resize, updates memory use semantics;
- Any request to decrease capacity thus **may be ignored**.

Algebraic type traits

Algebraic type traits: compile-time introspection of algebraic info

- grb :: is_associative< Operator >::value, `true` iff $(a \odot b) \odot c = a \odot (b \odot c)$;
- grb :: is_idempotent< Operator >::value, `true` iff $a \odot a = a$;
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These are all **compile-time constant** (through C++11 constexpr):

- similar to the standard C++11 *type traits*.

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Algebraic type traits help

- detect programmer errors,
- decide which optimisations are applicable, and
- reject expressions without recipe for auto-parallelisation.

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For example, **algebraic type traits prevent**

- creating a monoid from non-associative operator;
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without requiring programmer knowledge or intervention.

Ex.: Y & Bisseling '10; Y & Roose '14; Y, Bisseling, Roose, Meerbergen '14; Y, Roose, Meerbergen '14; ...