ALTO: Mode-Agnostic Linearized Storage for High-Performance Sparse Tensor Computations

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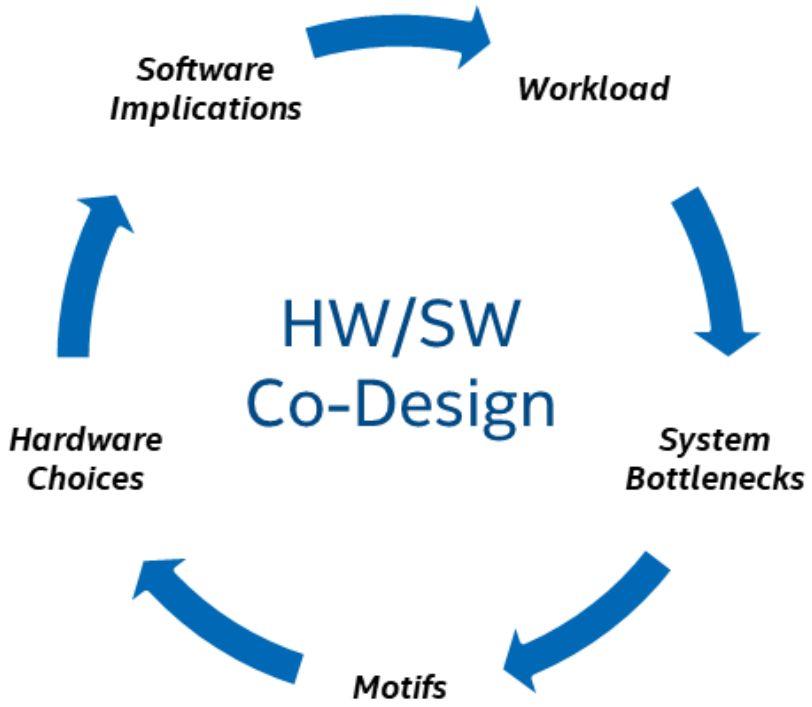
Teresa Ranadive,† Fabrizio Petrini,* Jee Choi ‡

Intel labs,* Laboratory for Physical Sciences (LPS),† University of Oregon‡

Intel PCL

Our modus operandi:



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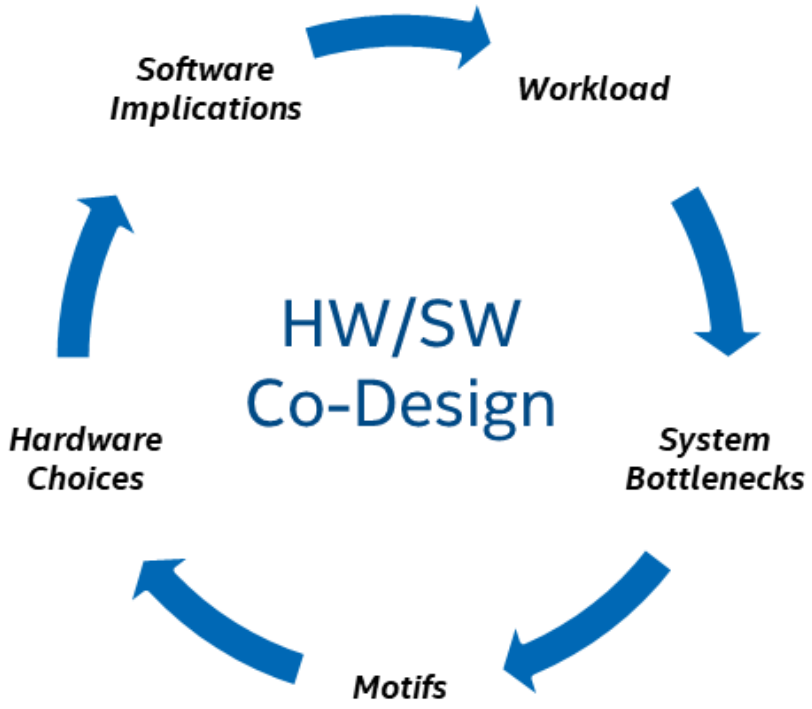

- **Application-driven** research to improve **efficiency** and **performance** of Intel's architectures and tools

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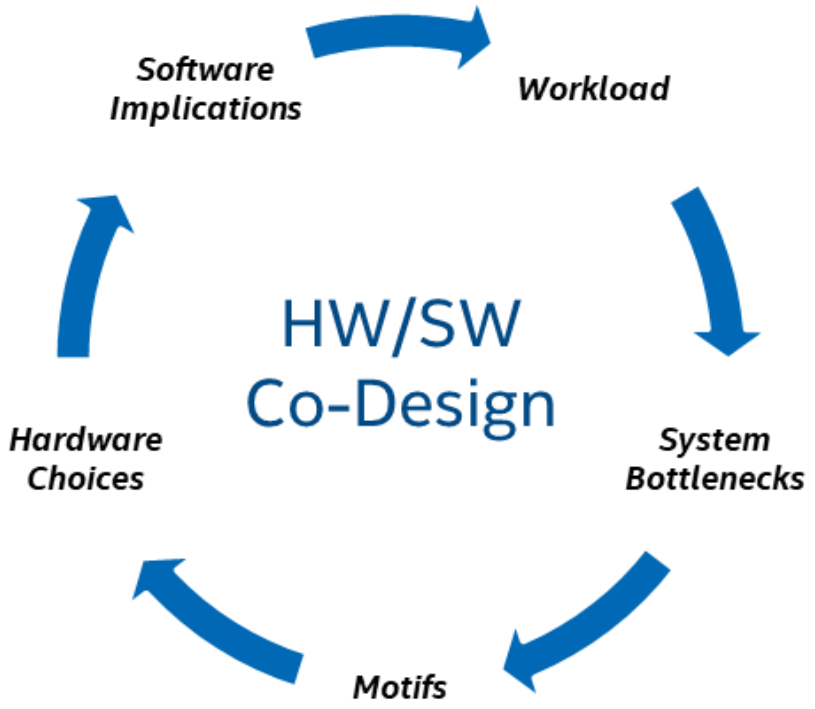

- **Application-driven** research to improve **efficiency** and **performance** of Intel's architectures and tools
- Target **domains**: machine learning (ML), high-performance computing (HPC), and data analytics

Outline

- Background & Motivation
  - State-of-the-art sparse tensor formats
- ALTO storage format
- Results & performance evaluation
- Conclusion

Motivation

- Many important application domains produce and manipulate massive amounts of **high-dimensional sparse data**

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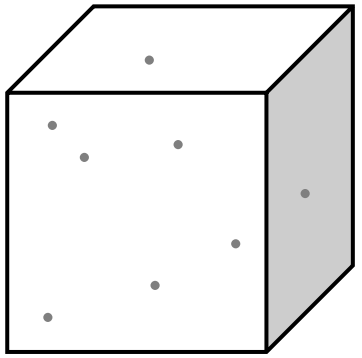

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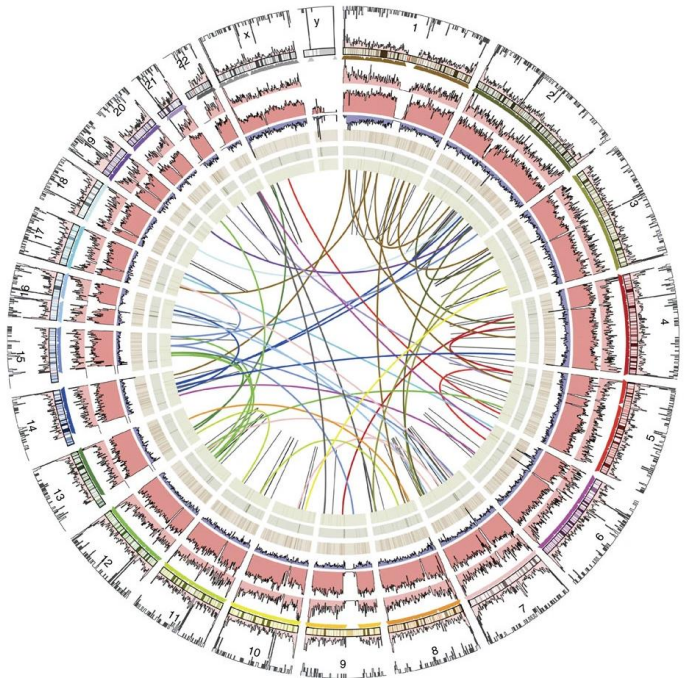

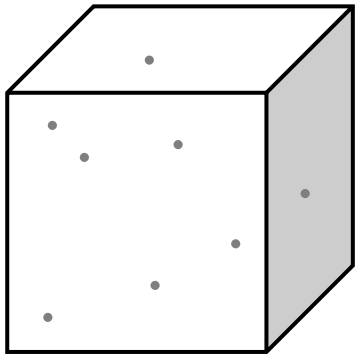

Source: Nature Communications and Scientific Reports, Wikimedia Commons

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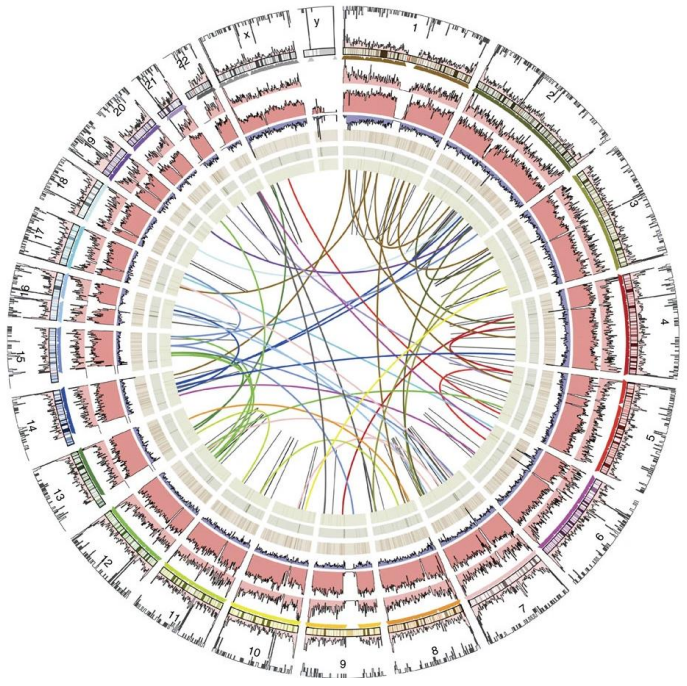

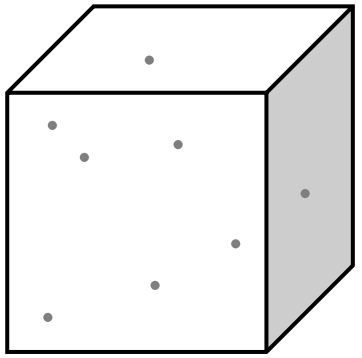

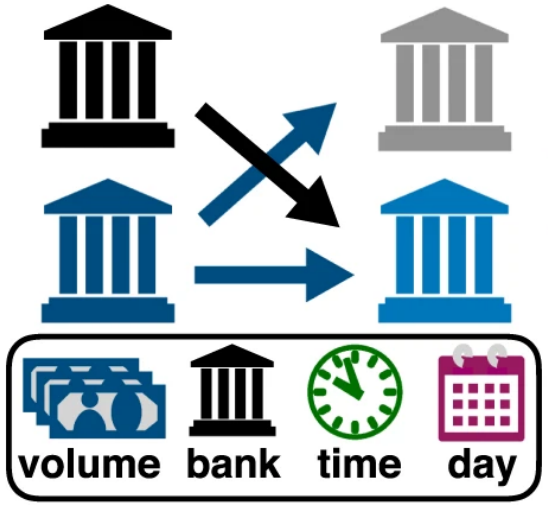

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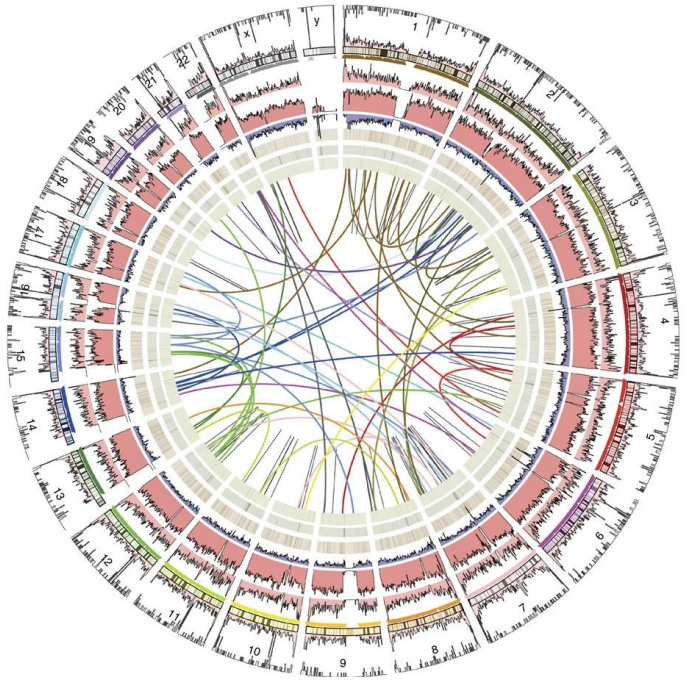

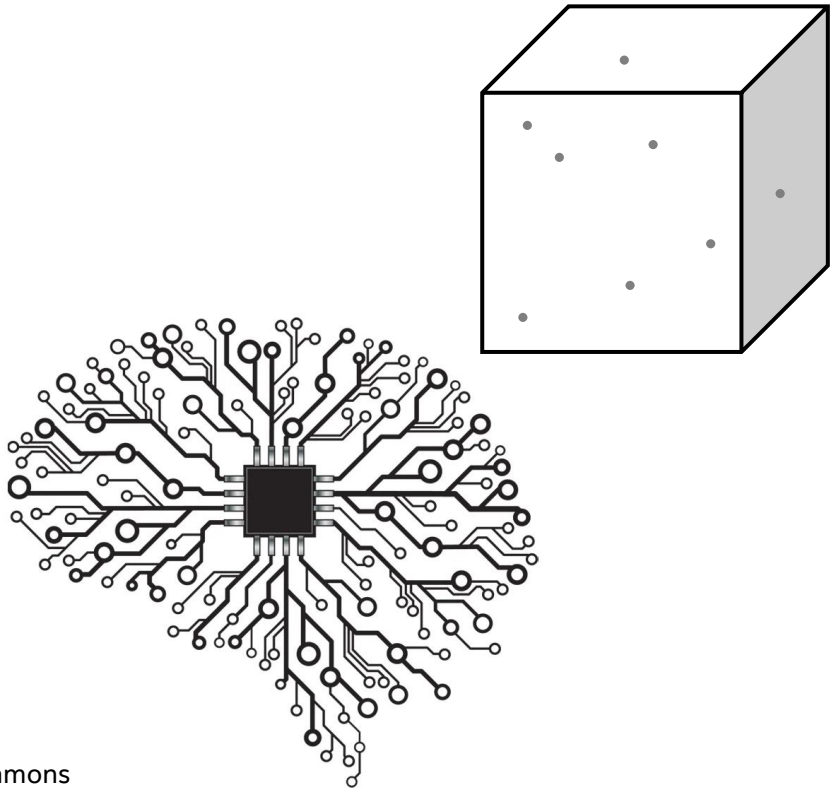

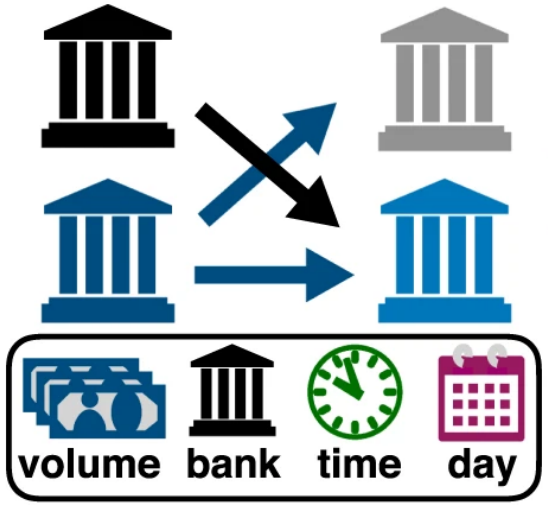

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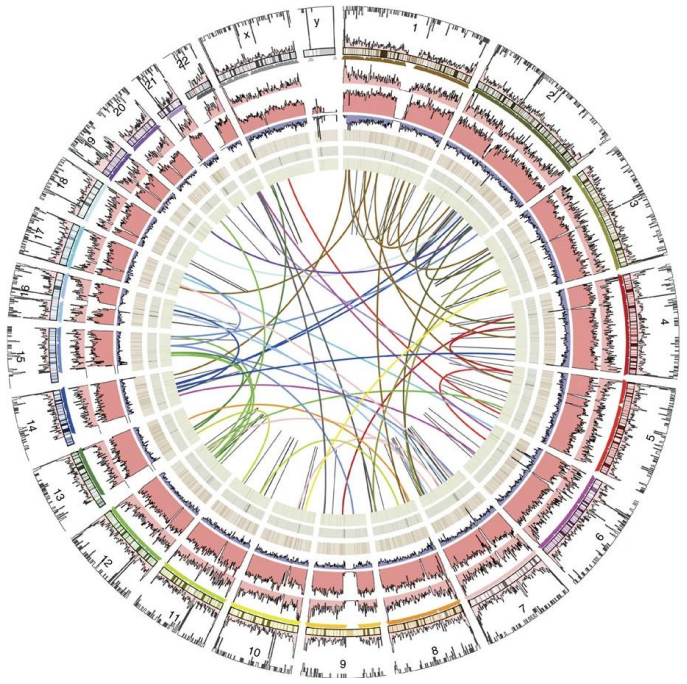

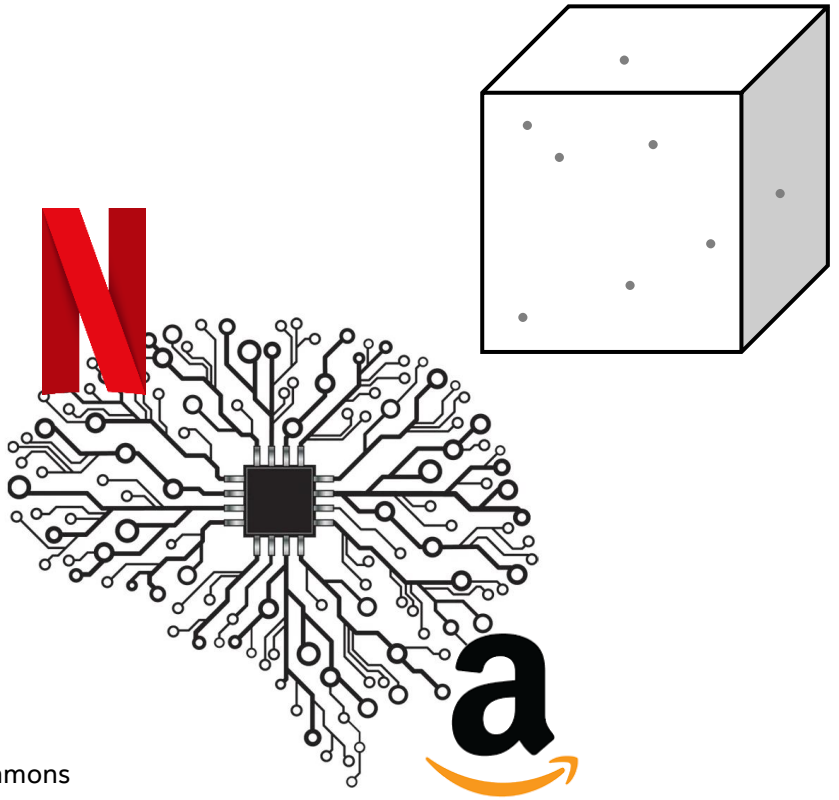

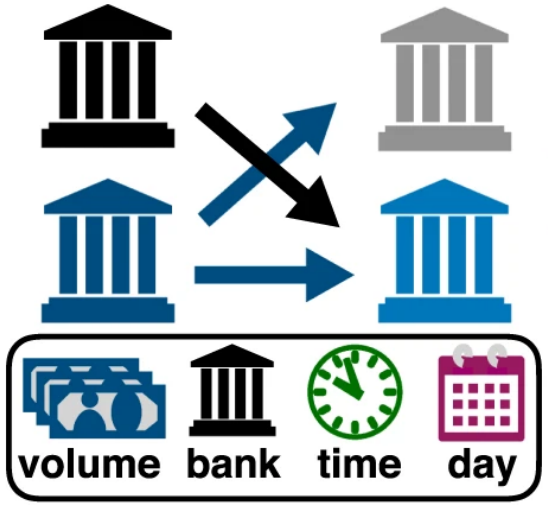

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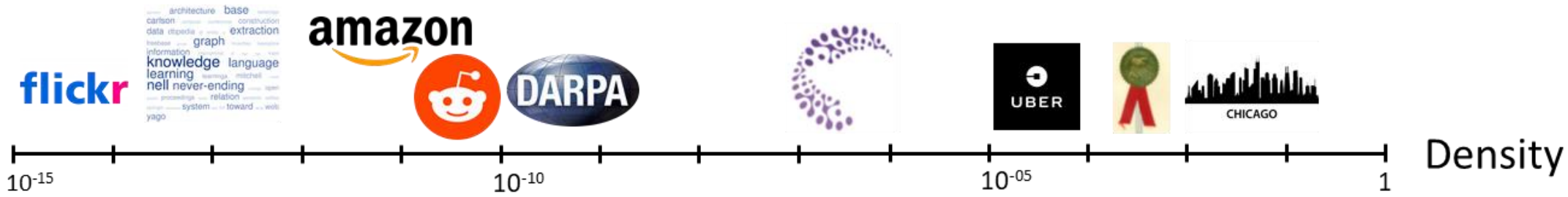
- High-dimensional datasets exhibit diverse shapes and data distributions as well as unstructured sparsity patterns



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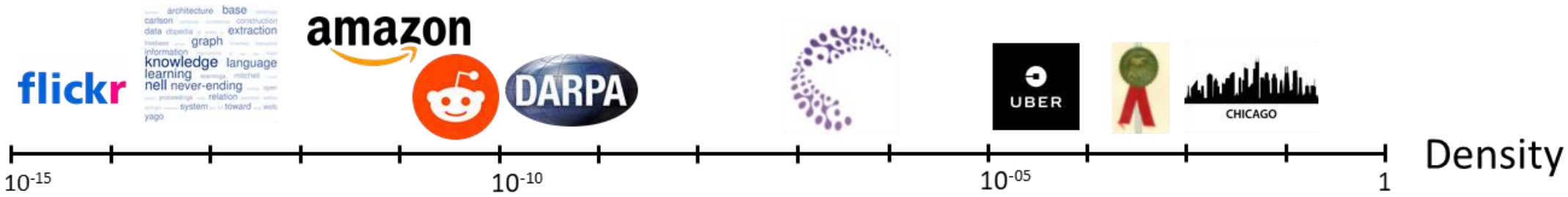
- High-dimensional datasets exhibit diverse shapes and data distributions as well as unstructured sparsity patterns
- Challenging to efficiently encode and represent



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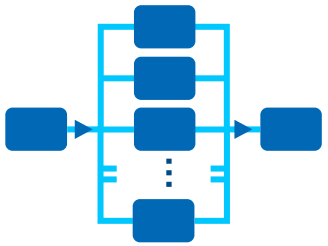

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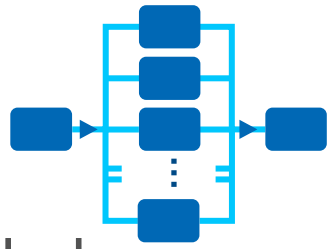
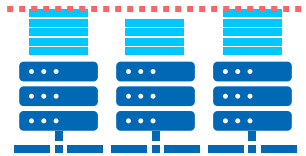

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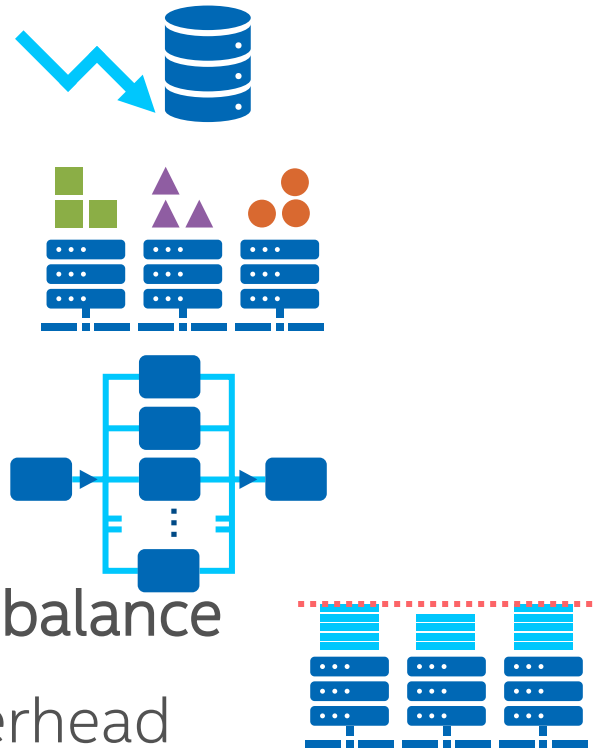

Oftentimes conflicting goals

Background

MTTKRP (Matricized Tensor Times Khatri-Rao product)

$$A = X_{(1)}(C \odot B)$$

- A key memory-intensive kernel in many sparse tensor computations

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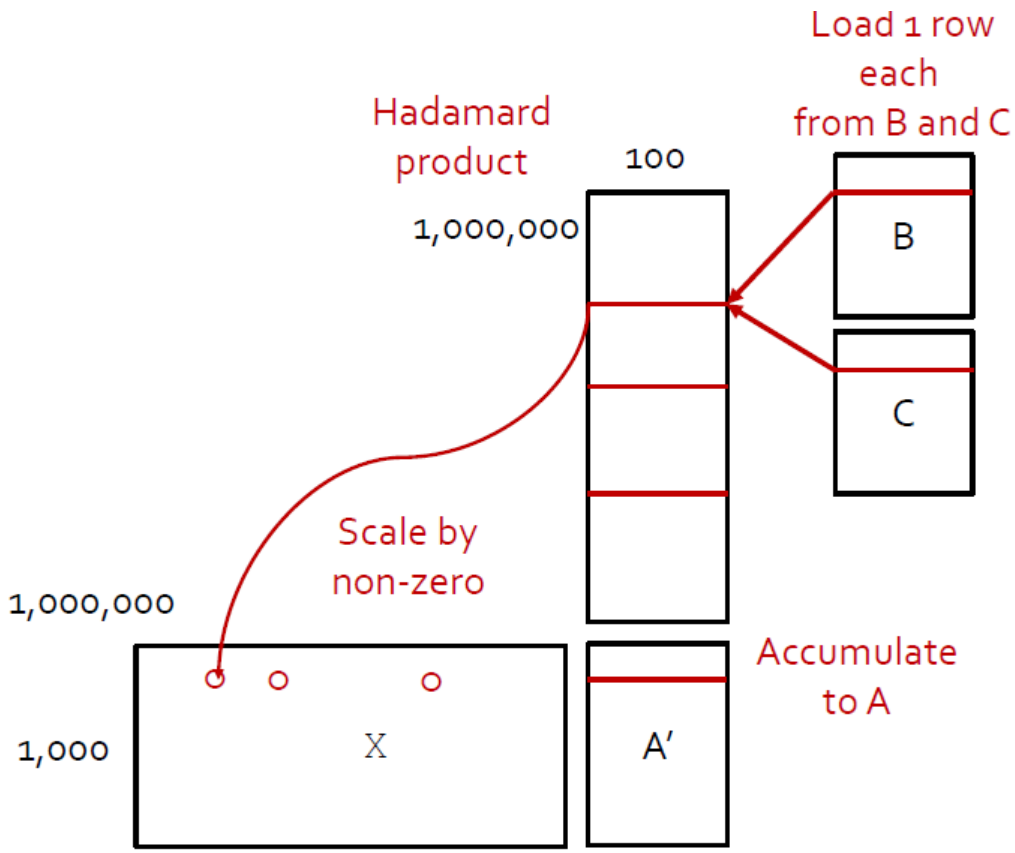

Source: Blocking Optimization Strategies for Sparse Tensor Computation, Jee Choi et al.

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- Challenge: hard to efficiently utilize the memory system for **all dimensions (modes)**

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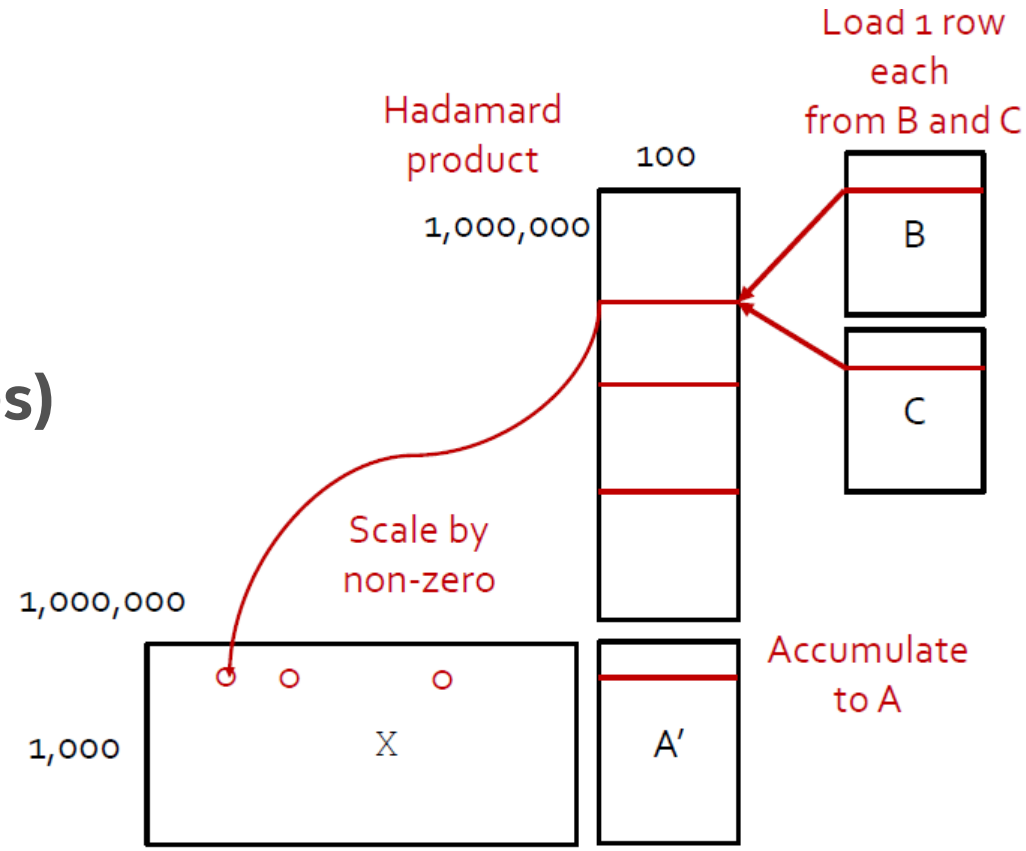

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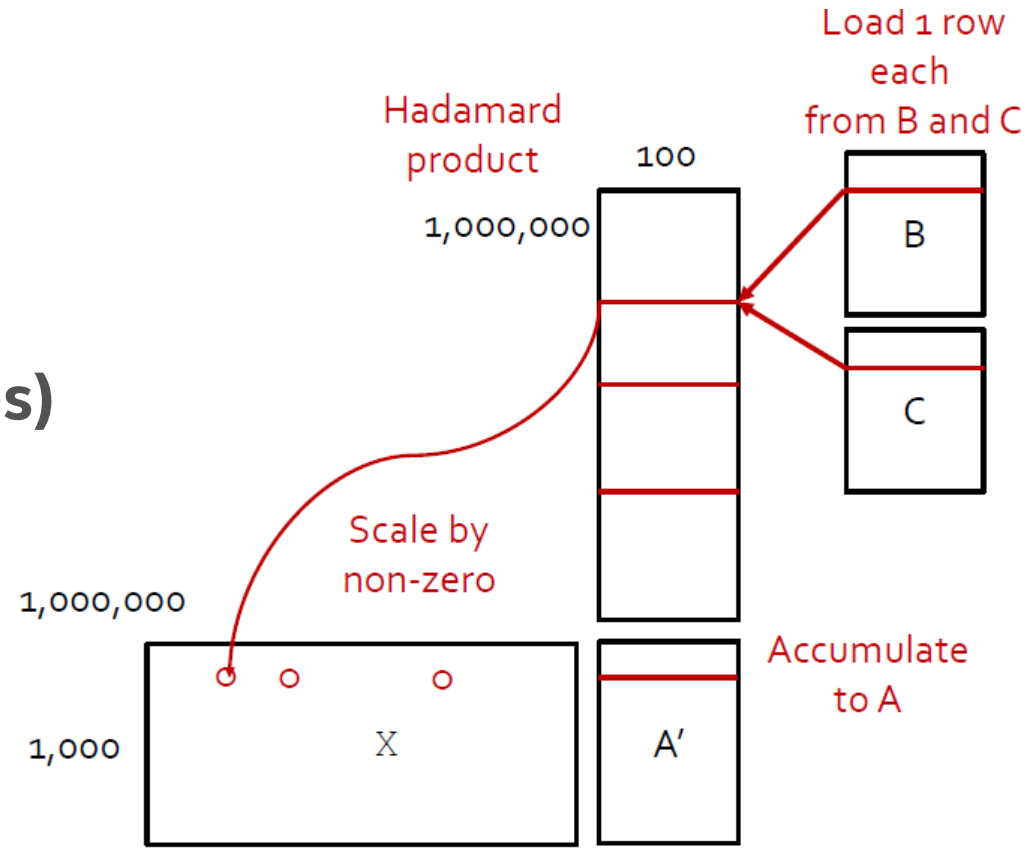
- Challenge: hard to efficiently utilize the memory system for **all dimensions (modes)**
  - **Different data locality patterns across modes**
  - **Update conflicts to factor matrices**
  - **Load imbalance across threads**



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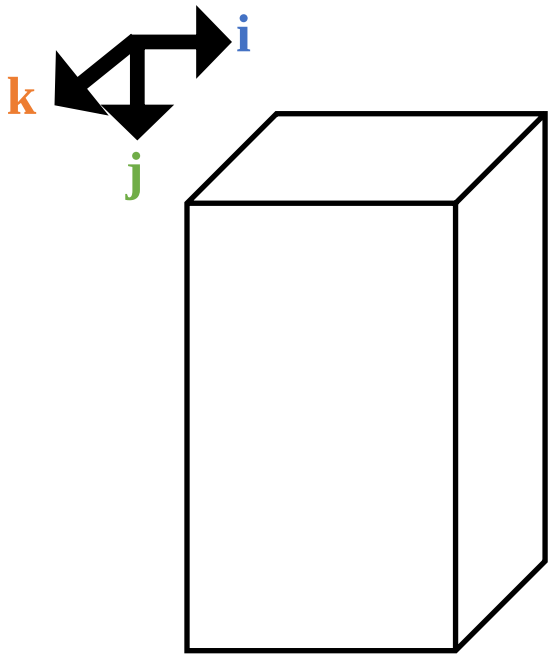
4x8x2 Tensor



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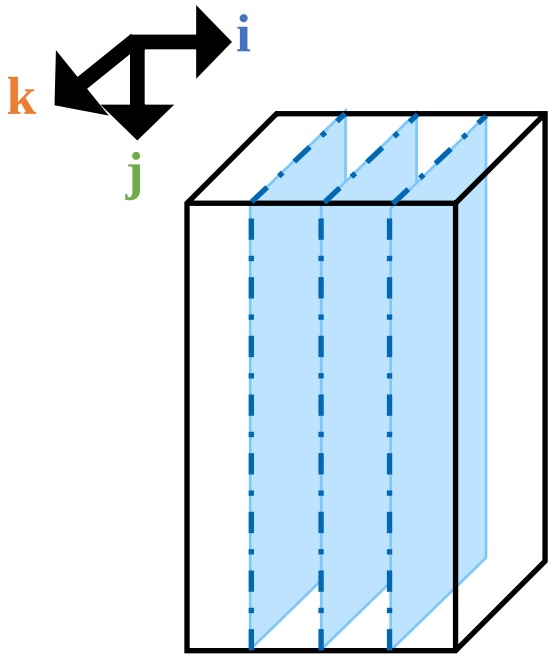

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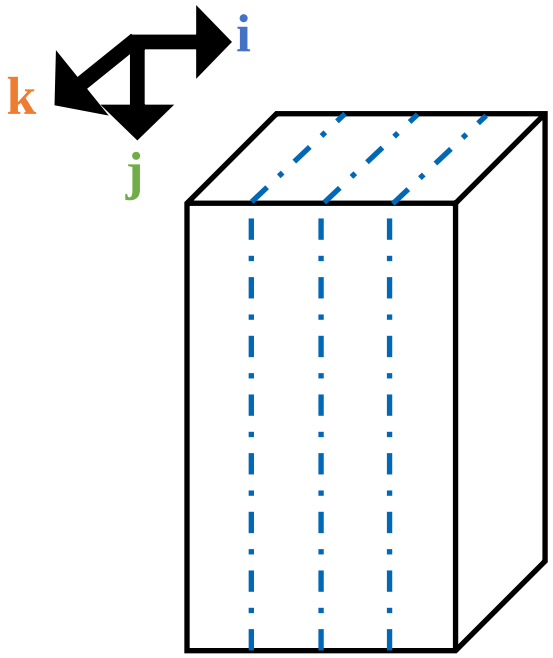

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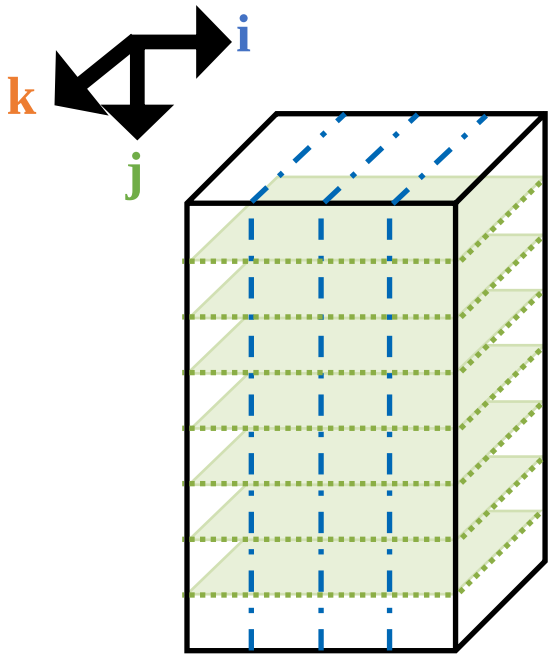

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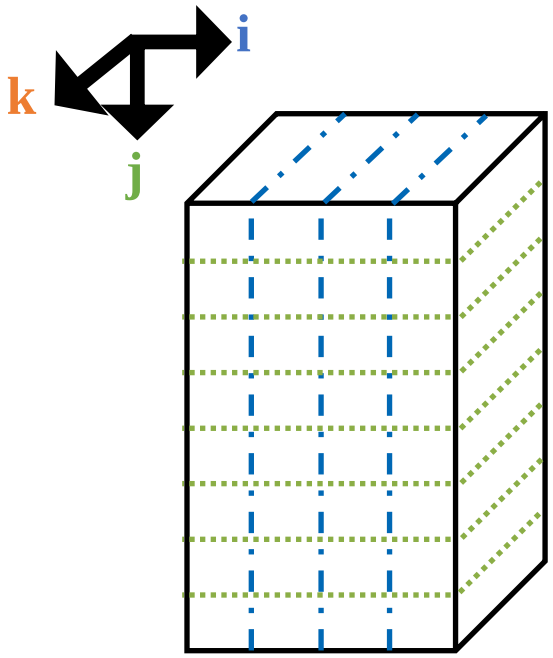

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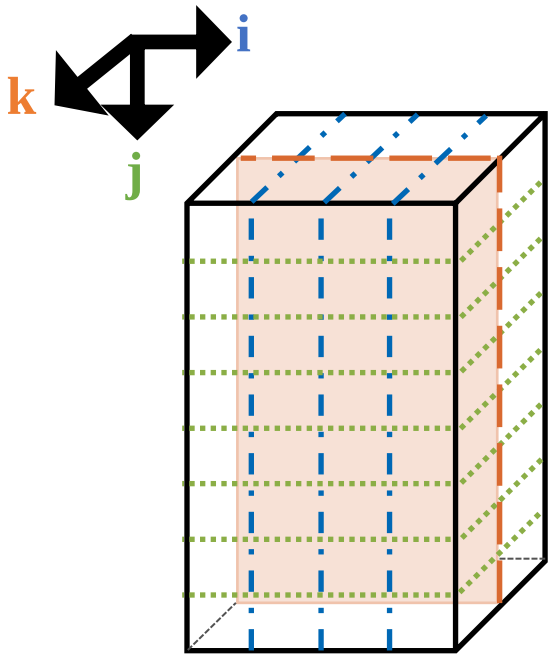

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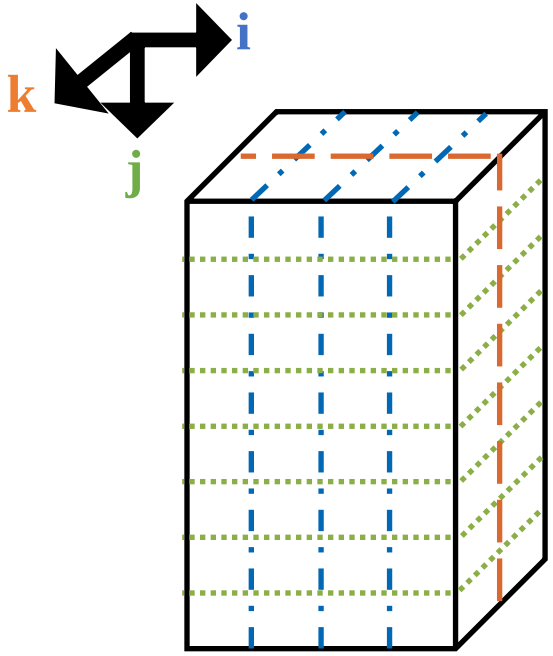

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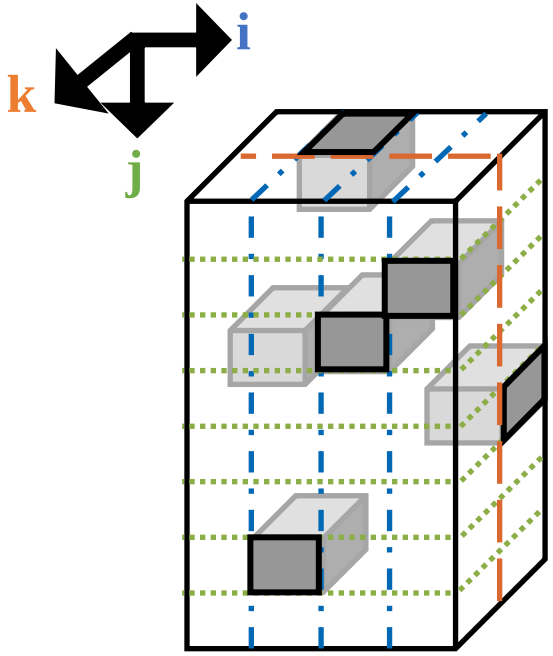

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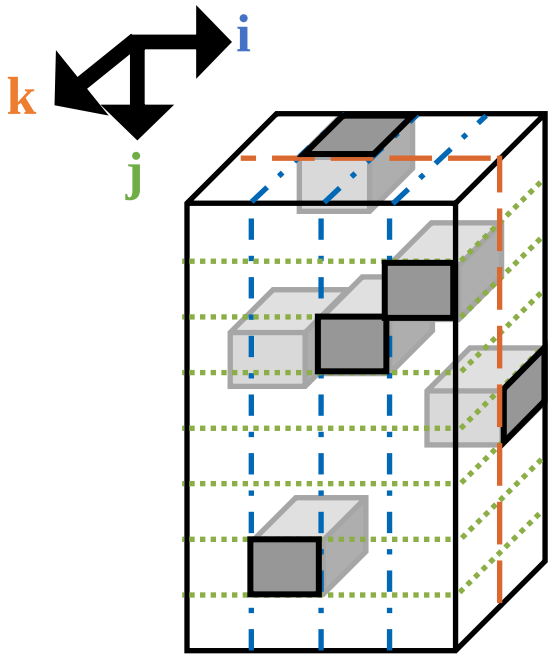
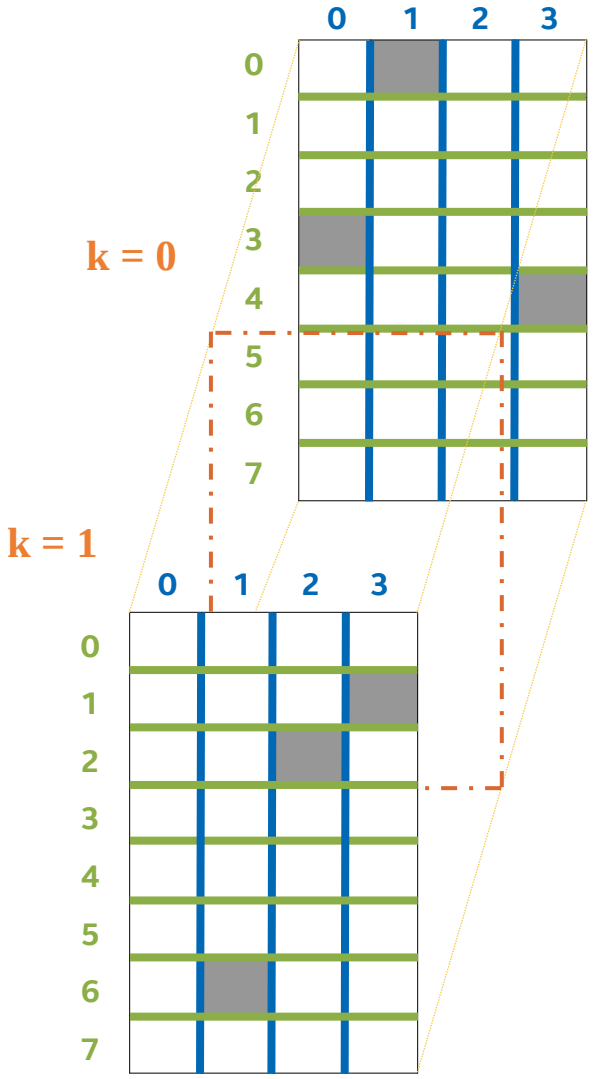
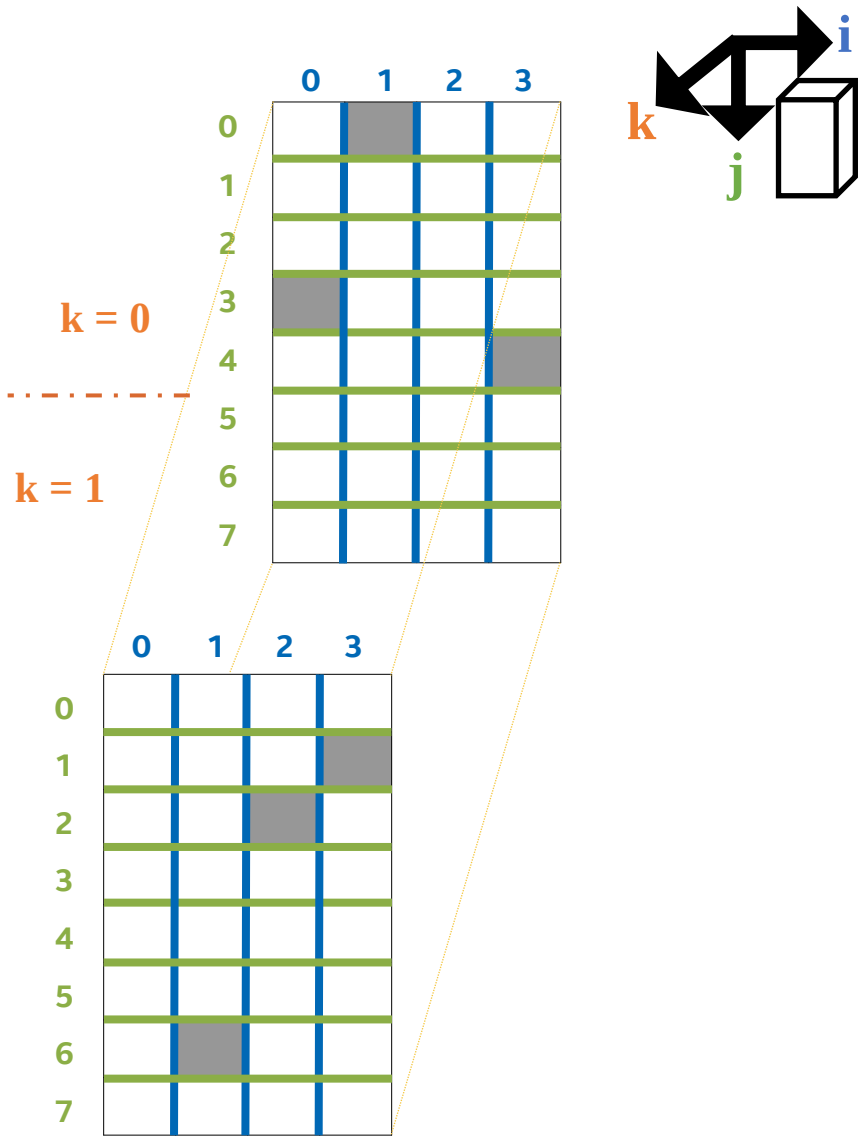




State-of-The-Art Formats

- Can be classified based on their encoding of the indexing metadata into:



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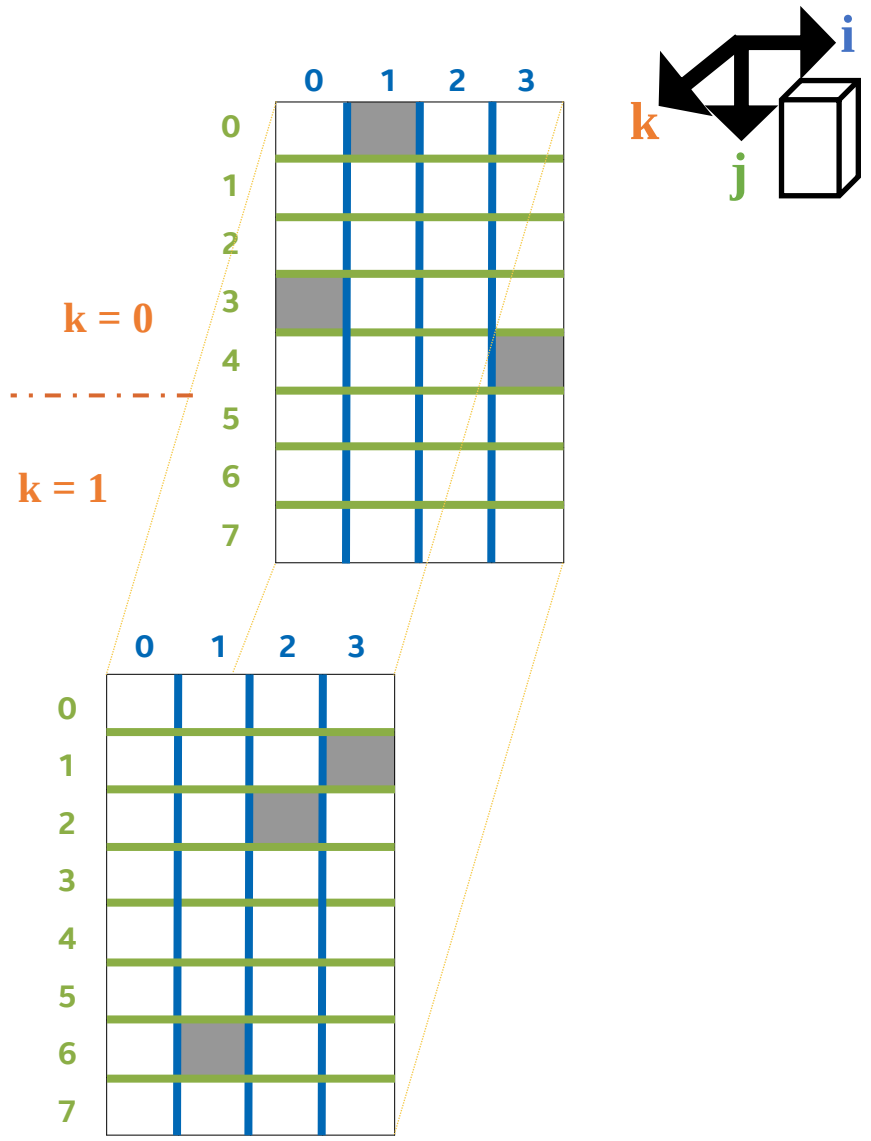

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- Can be classified based on their encoding of the indexing metadata into:
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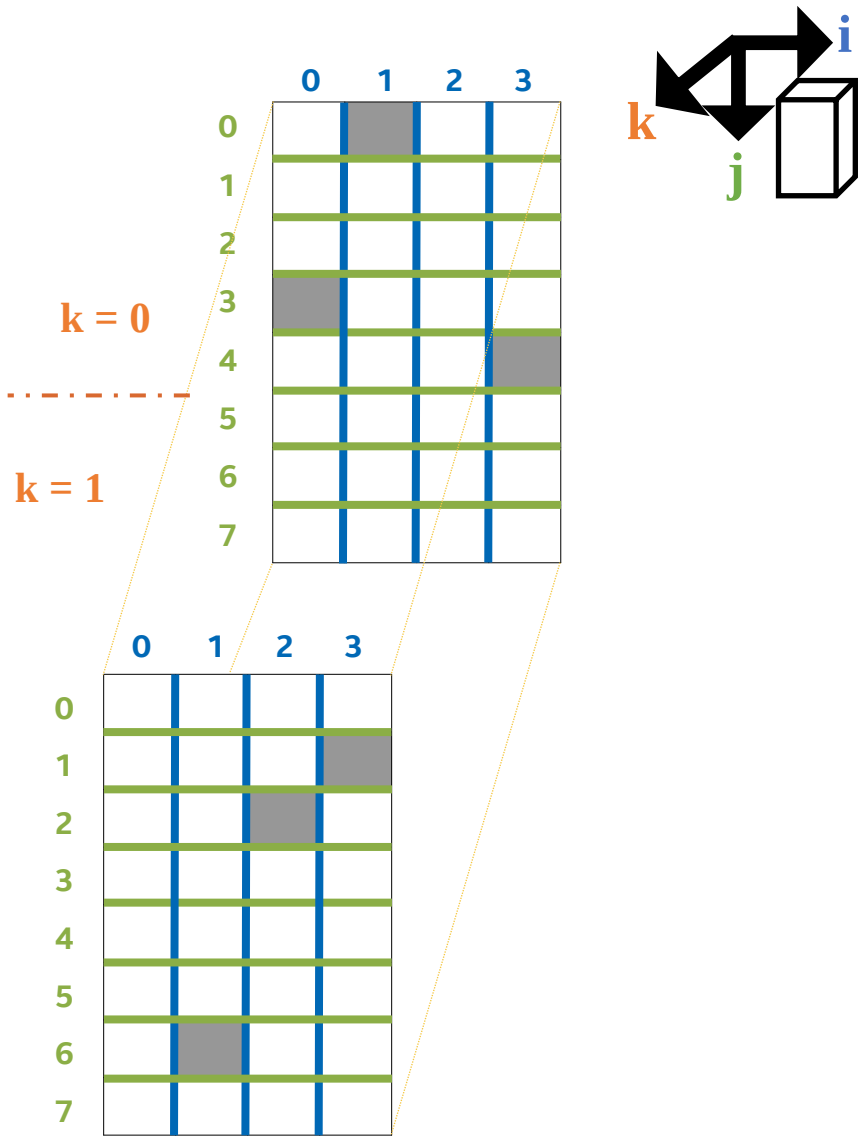

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- Can be classified based on their encoding of the indexing metadata into:
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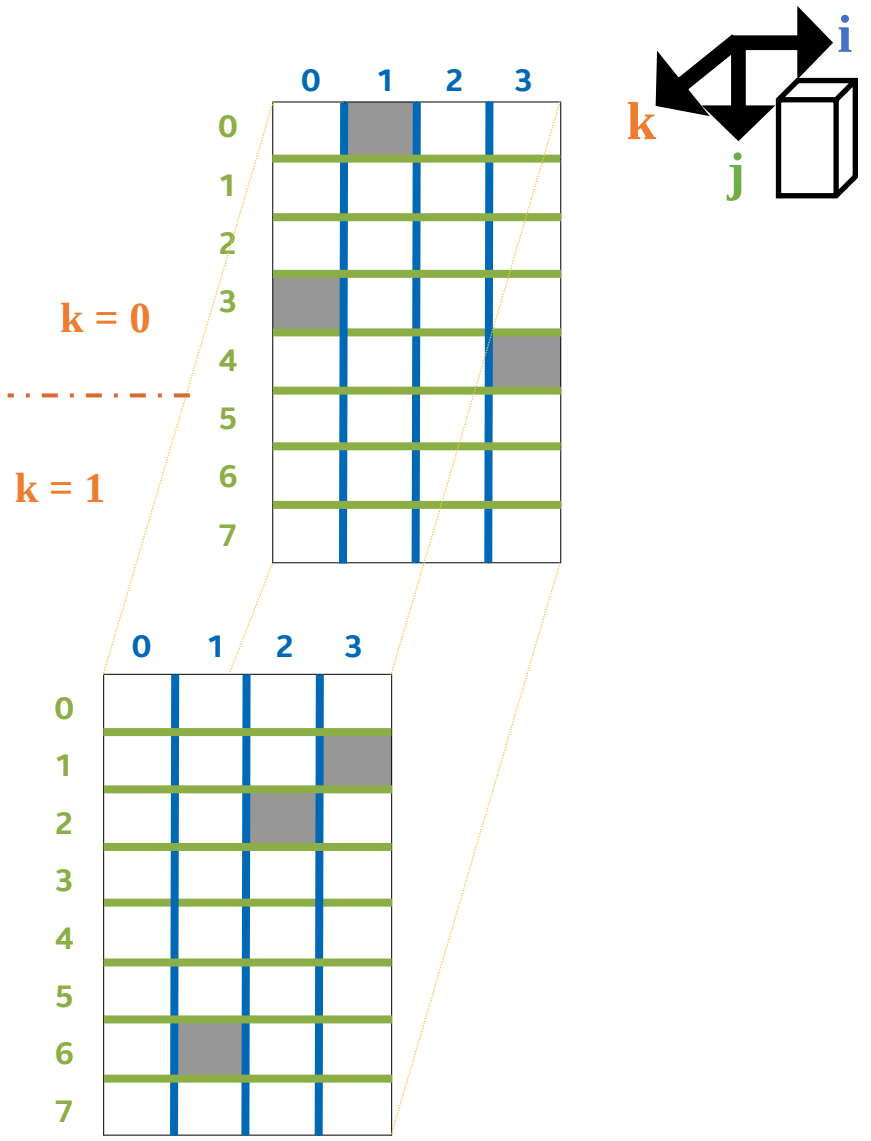
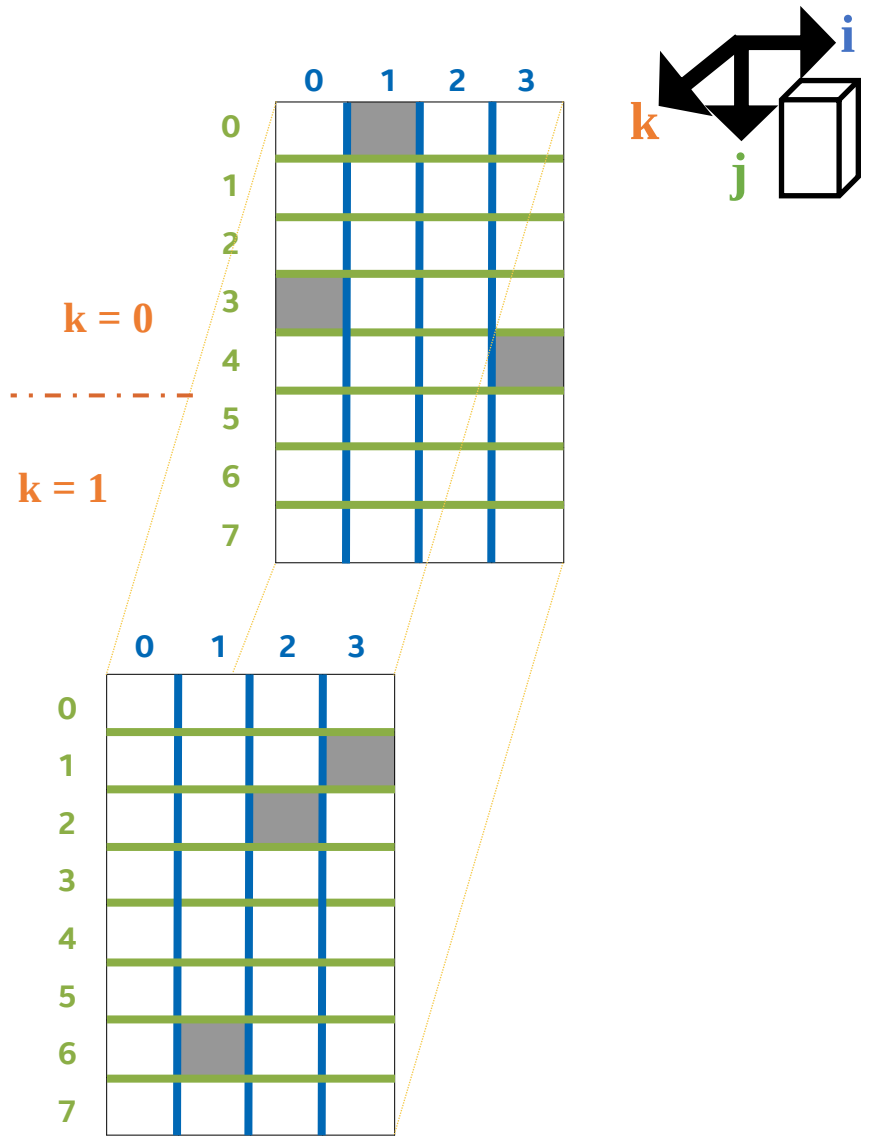
  3. Block-based formats



State-of-The-Art Formats

- COO (coordinate): list-based format

i	j	k
0	3	0
1	0	0
1	6	1
2	2	1
3	1	1
3	4	0



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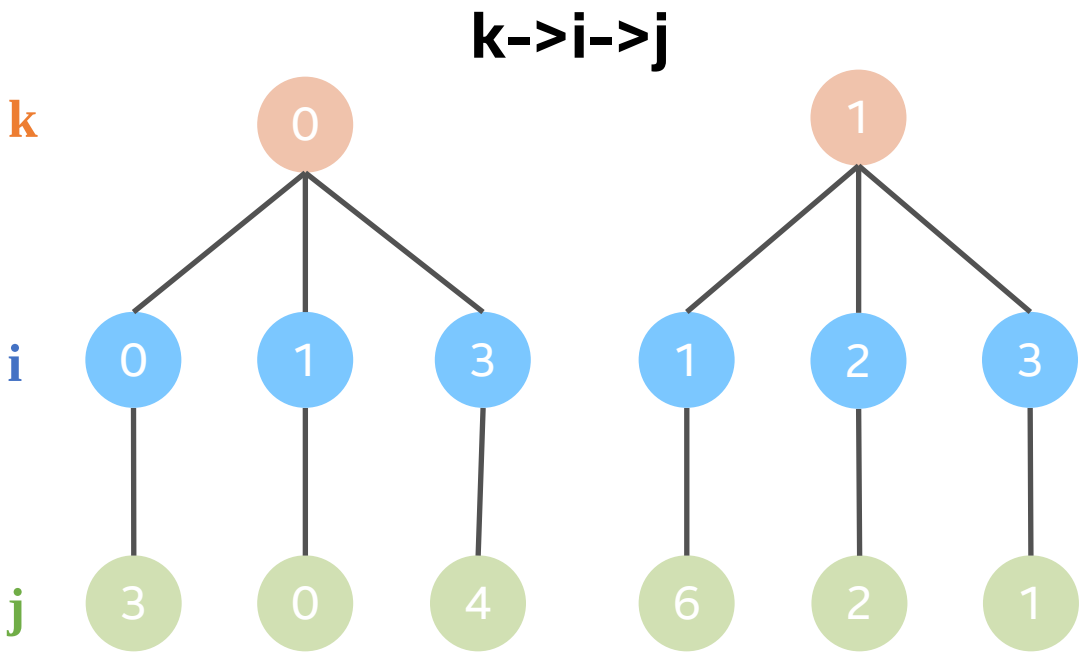
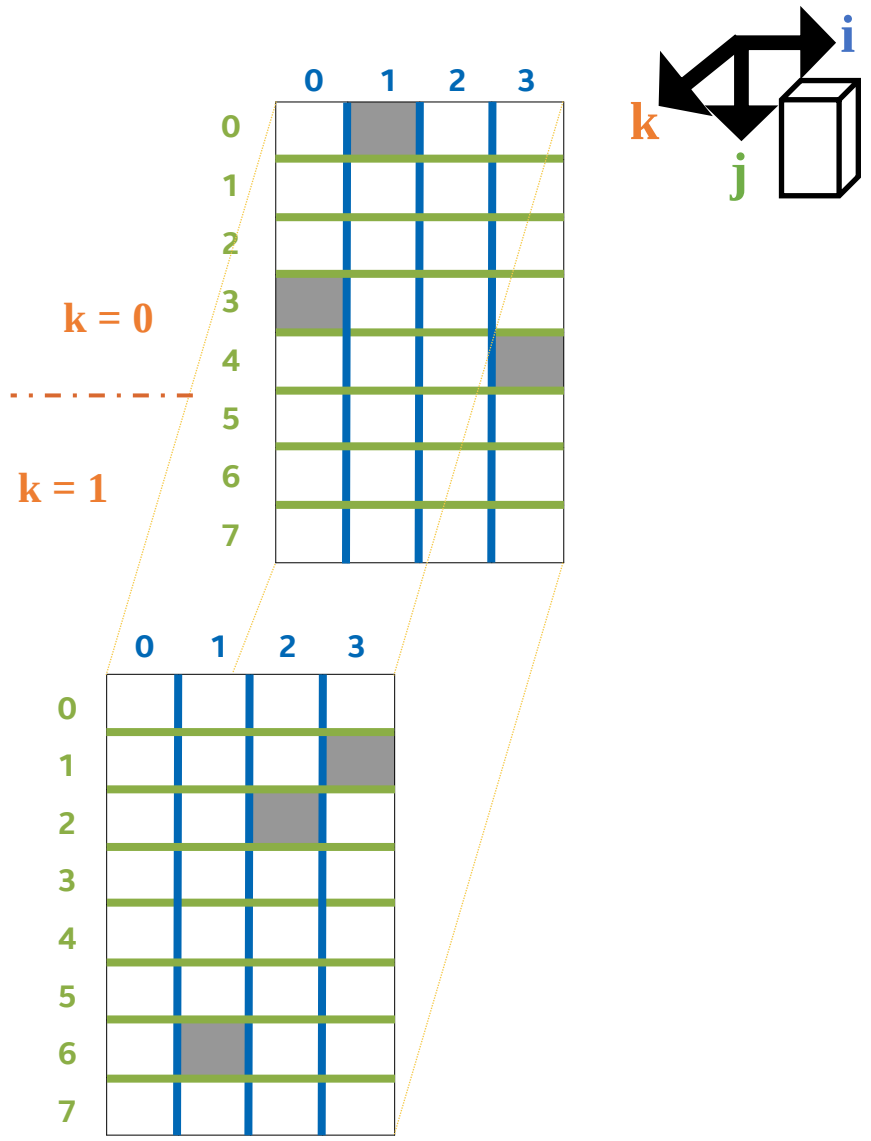
- CSF (compressed sparse fiber): **tree-based format**





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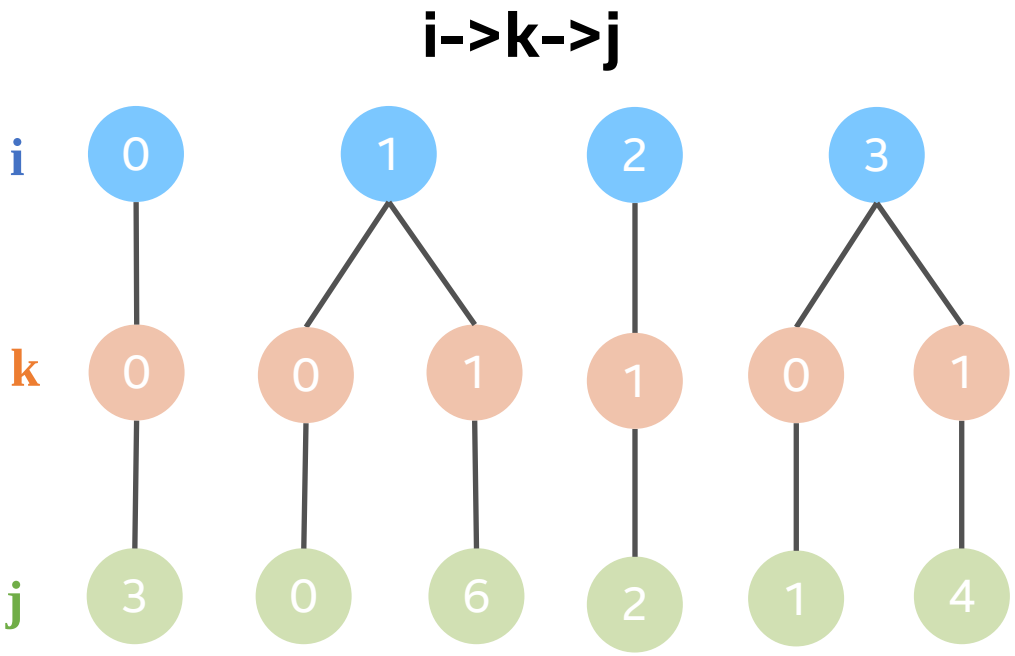

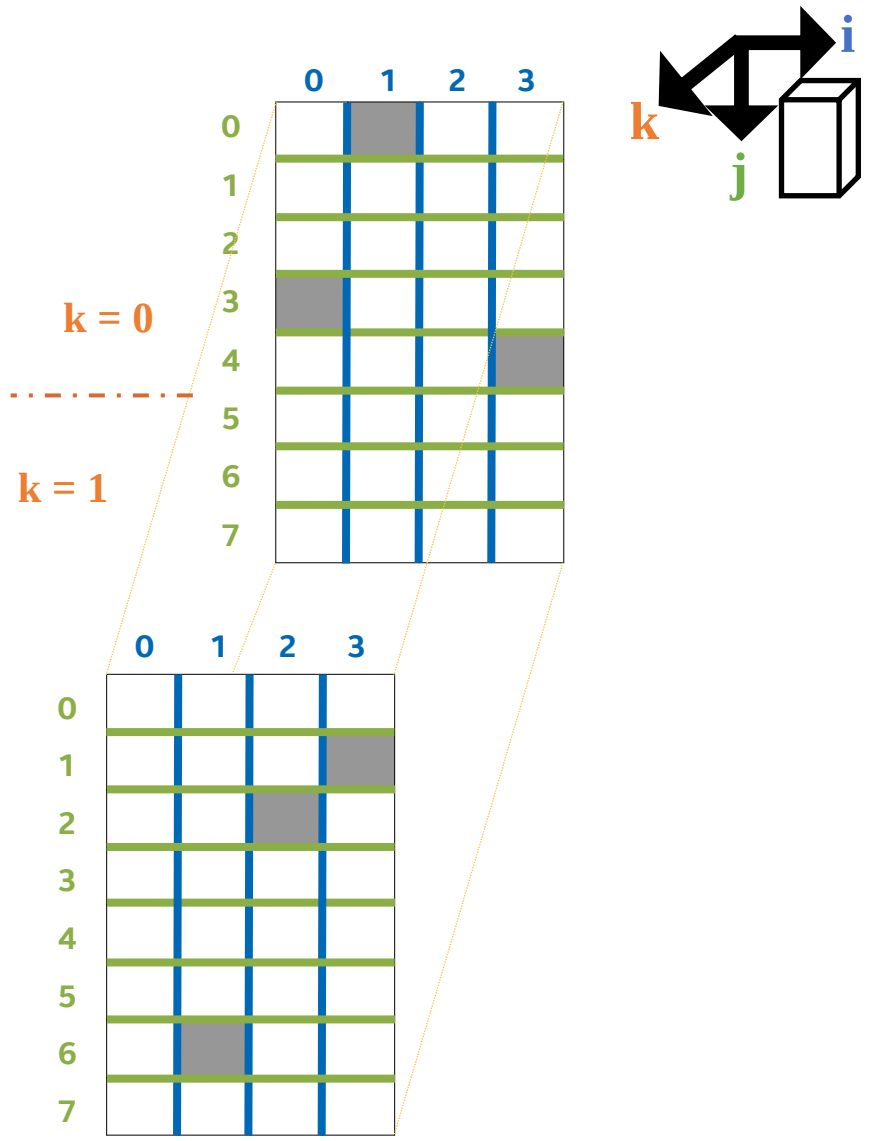

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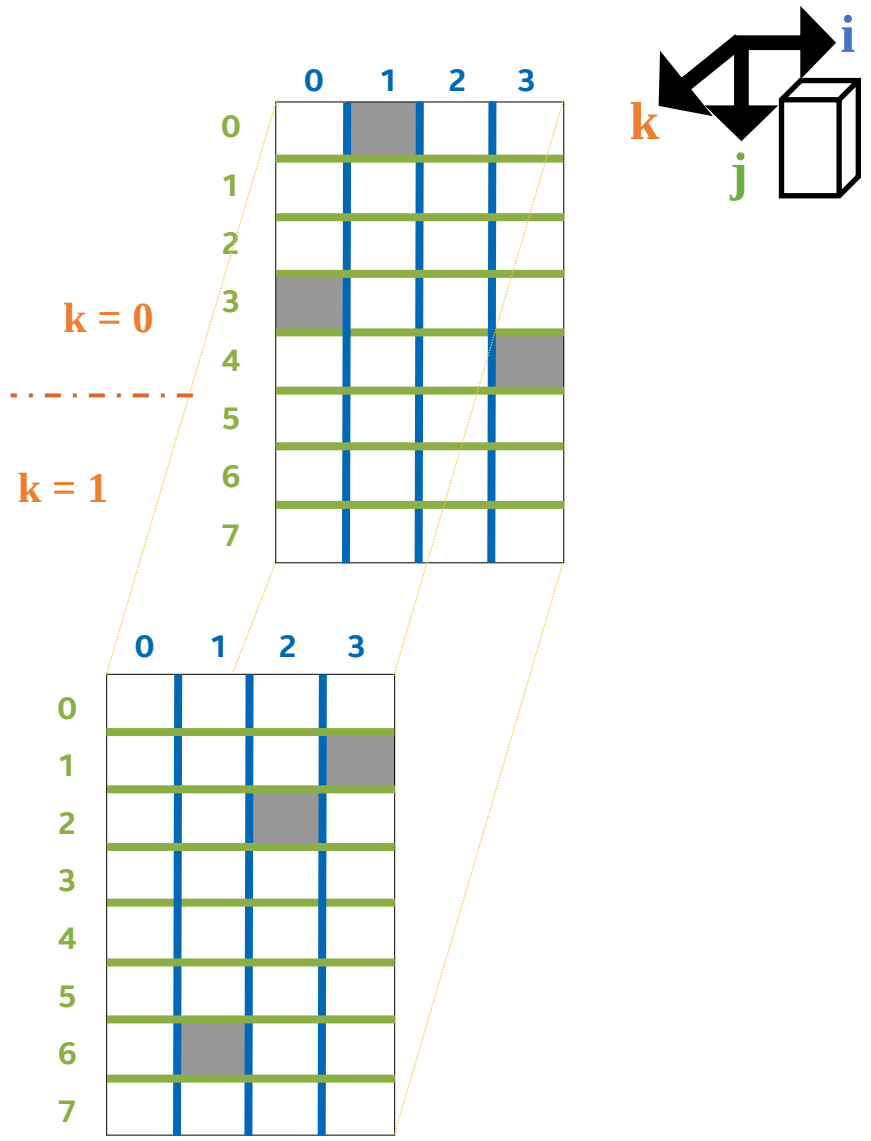
  - 6 possible representations for a 3D tensor



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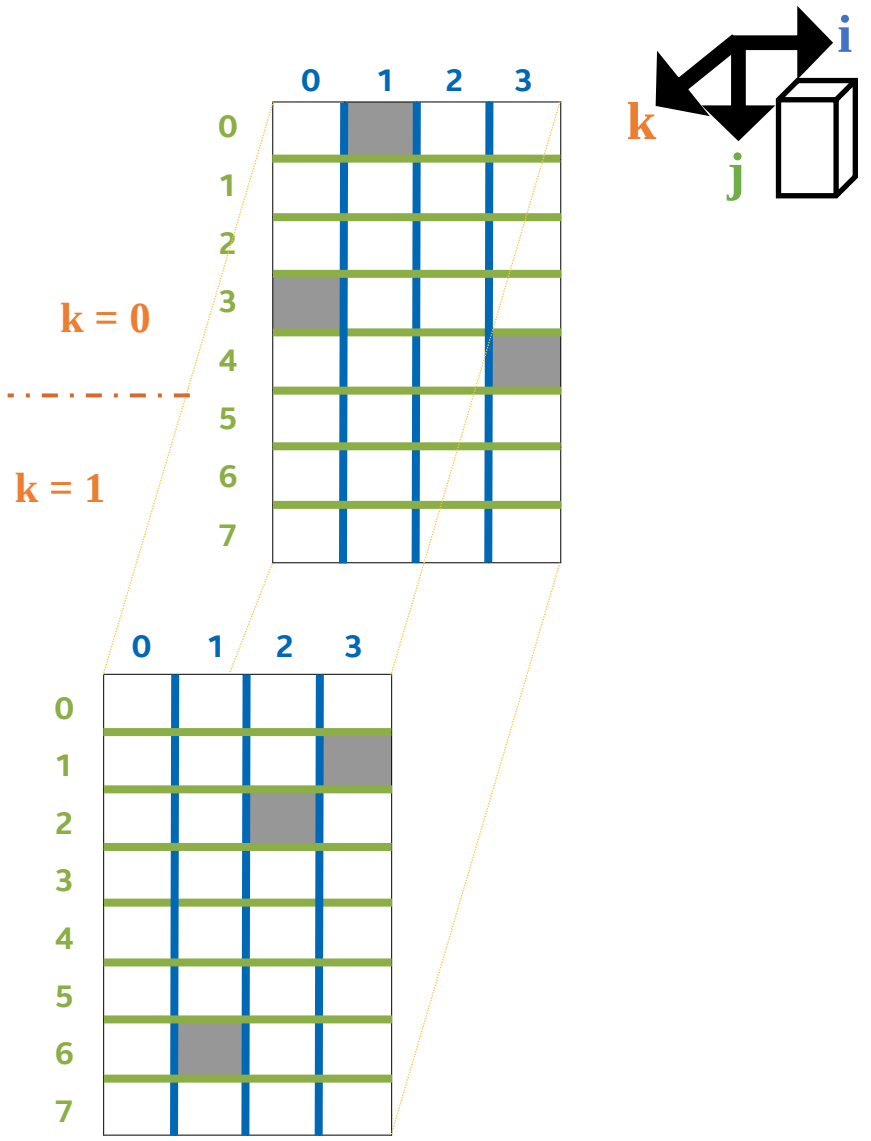

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- CSF (compressed sparse fiber): **tree-based format**
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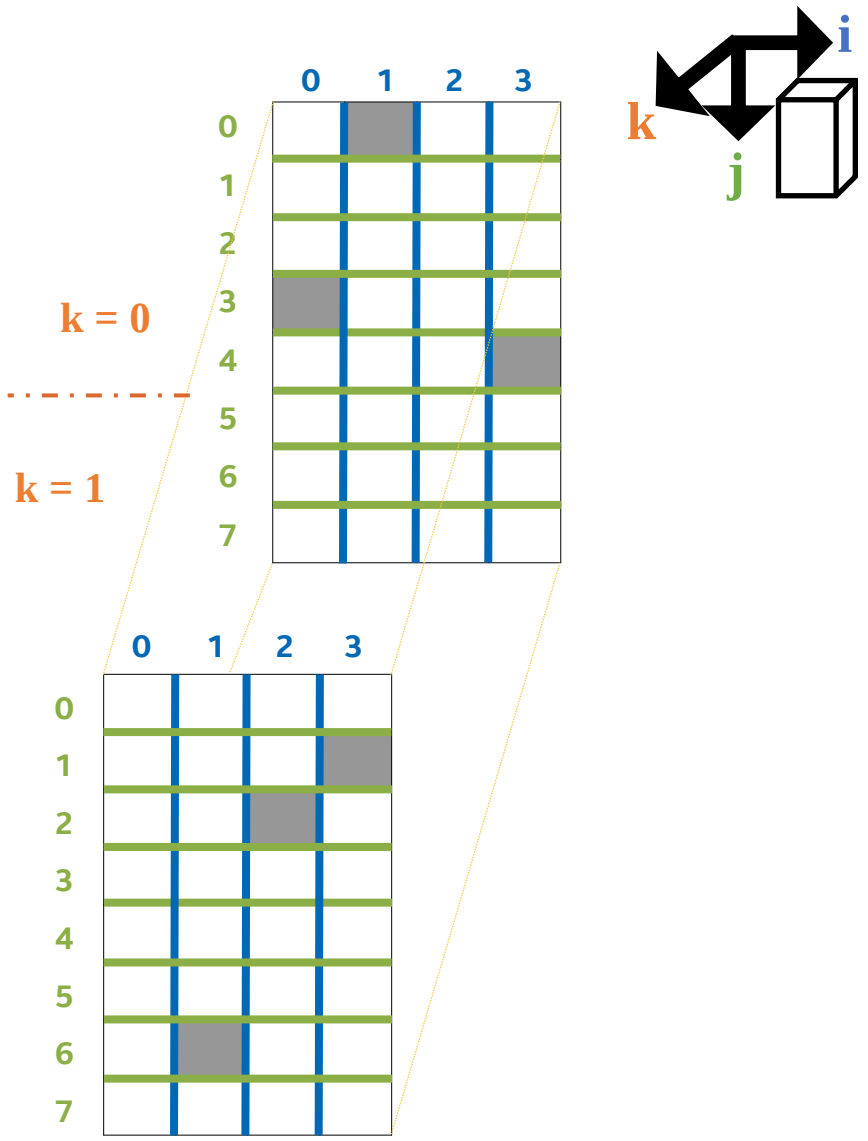
  - In practice, we only keep 3 representations
    1. i -> k -> j
    2. k -> i -> j
    3. j -> k -> i

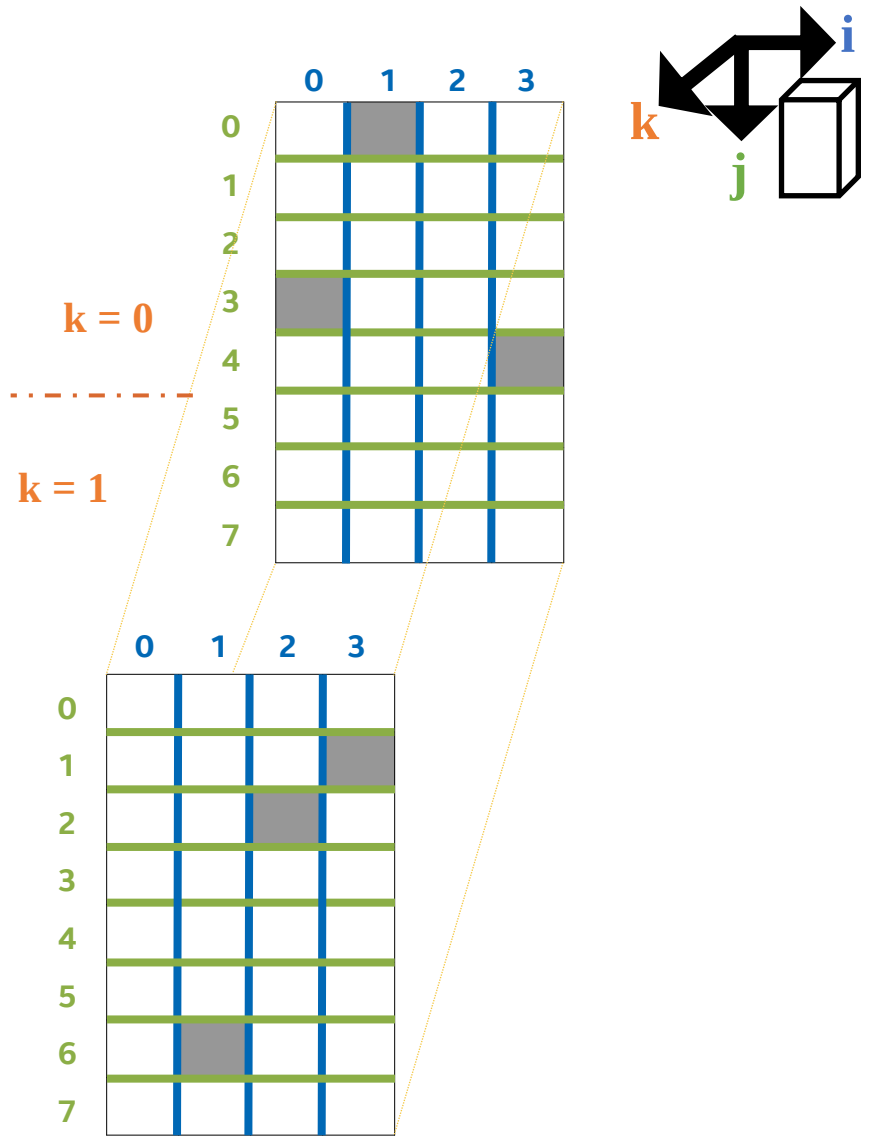


State-of-The-Art Formats

- HiCOO (hierarchical coordinate): **block-based format**
  - size(e-idx) << size(b-idx)

2X2X2

bptr	b_i	b_j	b_k	e_i	e_j	e_k
0	0	0	0	1	0	0
1	0	1	0	0	1	0
2	0	3	0	1	0	1
3	1	0	0	1	1	1
4	1	1	0	0	0	1
5	1	2	0	1	0	0



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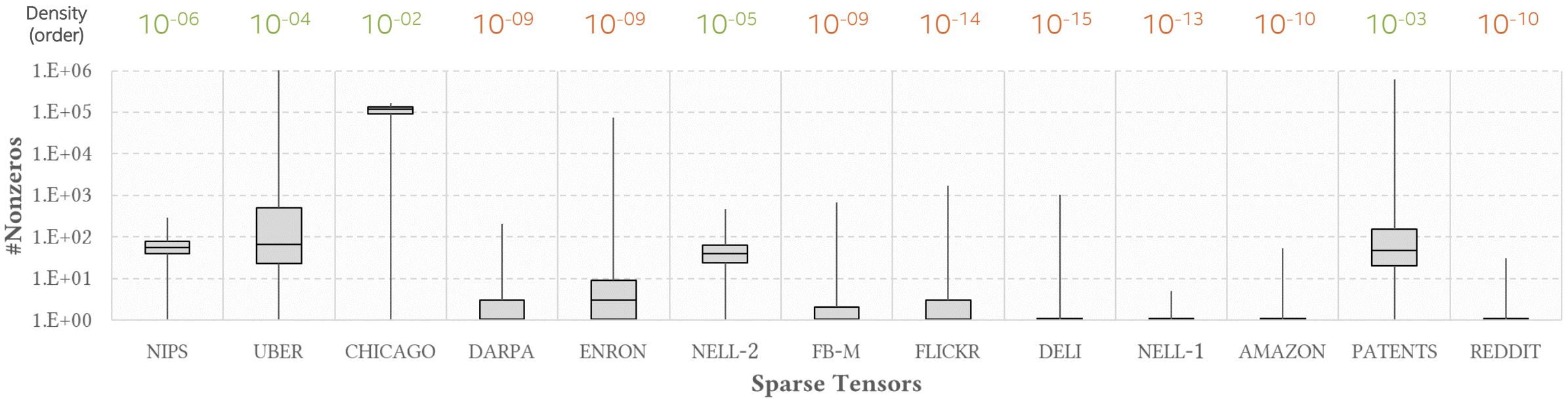
Spatial data distribution



A box plot of the data (nonzero elements) distribution across multi-dimensional blocks (subspaces). The multi-dimensional subspace size is 128^N, where N is the number of dimensions (modes).

State-of-The-Art Formats

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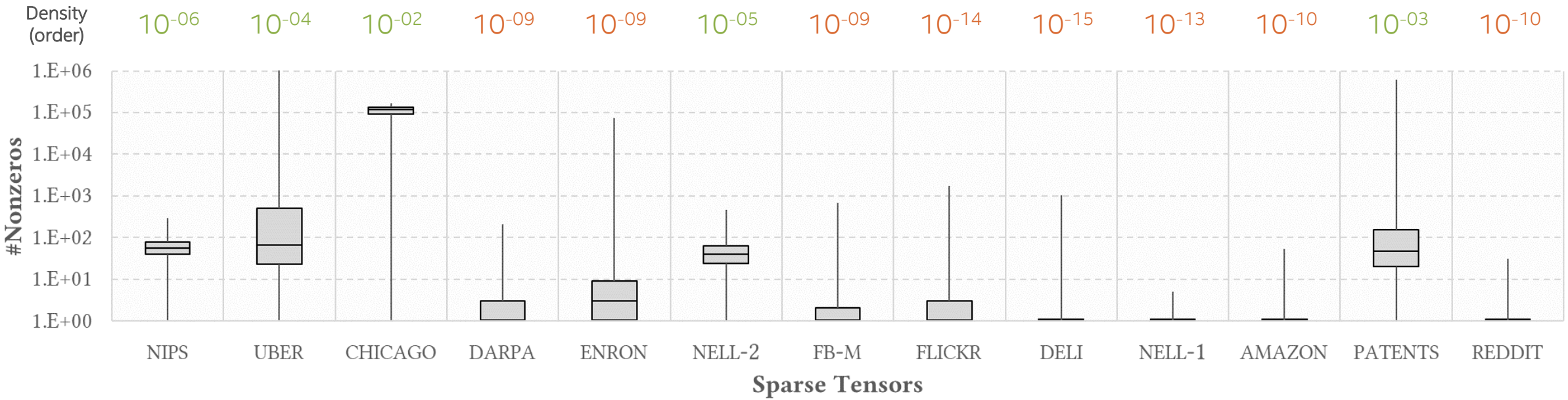

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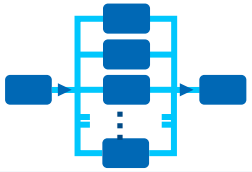
increased sparsity → inefficient memory usage

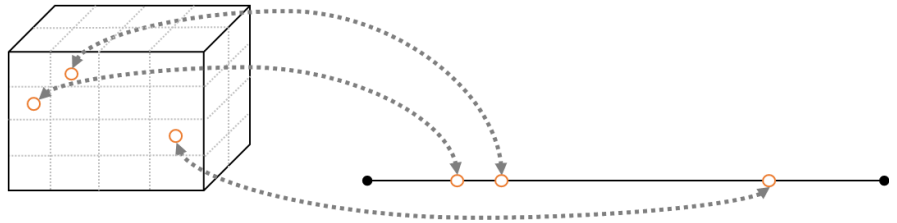
State-of-The-Art Formats


Formats	Granularity	Mode Orientation	Data Locality	Parallelism	Load balance
List-based (COO)	NNZ element	Mode-agnostic	NA	Suffer from conflicts	Maximized
Tree-based (CSF)	Compressed sparse fiber	Mode-specific	Improved for specific mode	Improved for specific mode	Work imbalance (especially in short modes)
Block-based (HiCOO)	Compressed block	Mode-agnostic	Improved	Suffer from conflicts	Work imbalance across blocks

Adaptive Linearized Tensor Order (**ALTO**)

- A new storage format for high performance and scalable sparse tensor computations
- Maps a multi-dimensional space into a linear space, and vice versa, using an adaptive (**data-aware**) encoding scheme



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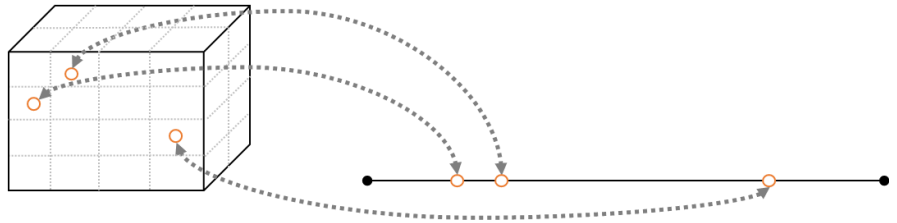

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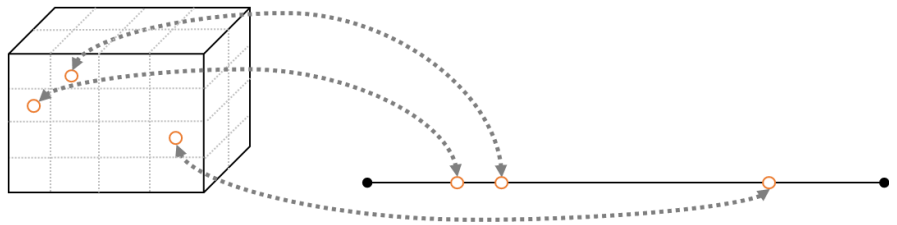

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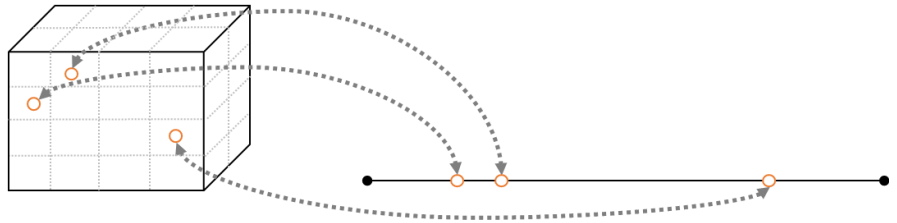

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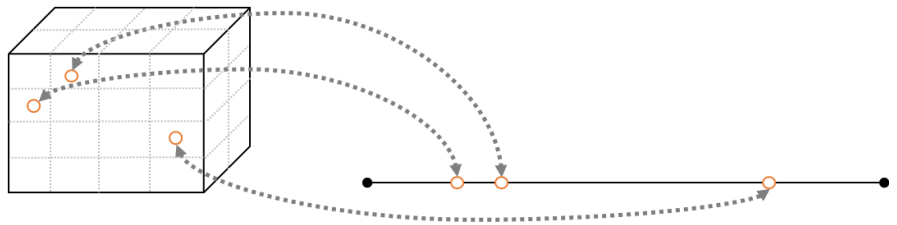

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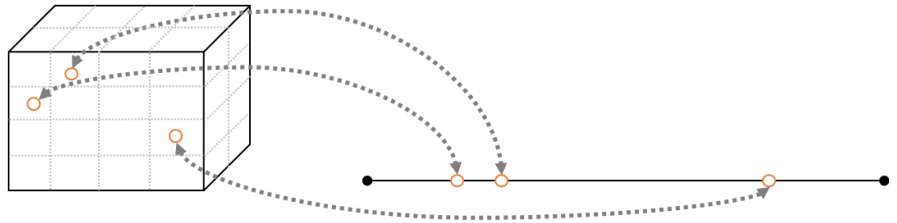
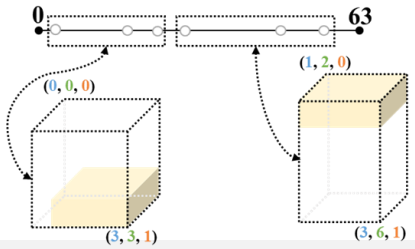
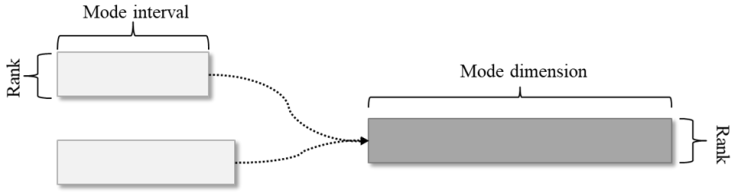
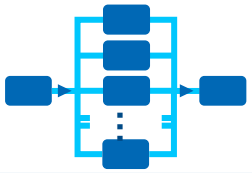


- **Data-aware** synchronization to resolve conflicting updates (writes) across threads





Adaptive Linearized Tensor Order (ALTO)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Adaptive Linearized Tensor Order (ALTO)

- ALTO **outperforms** an oracle that selects the best state-of-the-art format in terms of the parallel performance and tensor storage

- **Platform**: an Intel Xeon Platinum 8280 (CLX) w/ 2x28 cores
- **Mode-agnostic formats**: coordinate (COO) and hierarchical COO (HiCOO)
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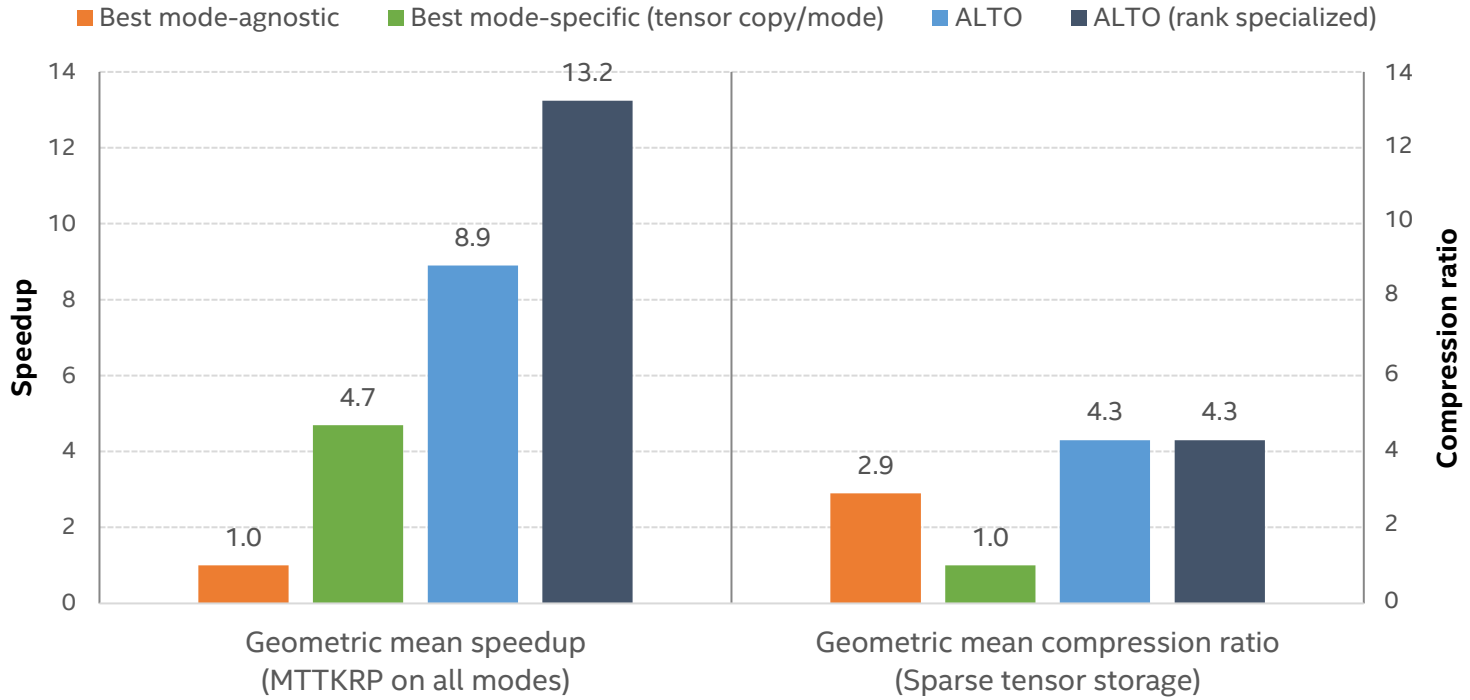

"ALTO: Adaptive Linearized Storage of Sparse Tensors", Ahmed Helal, Jan Laukemann, Fabio Checconi, Jesmin Jahan Tithi, Teresa Ranadive, Fabrizio Petrini, Jee Choi, **accepted** to ACM International Conference on Supercomputing (ICS), 2021

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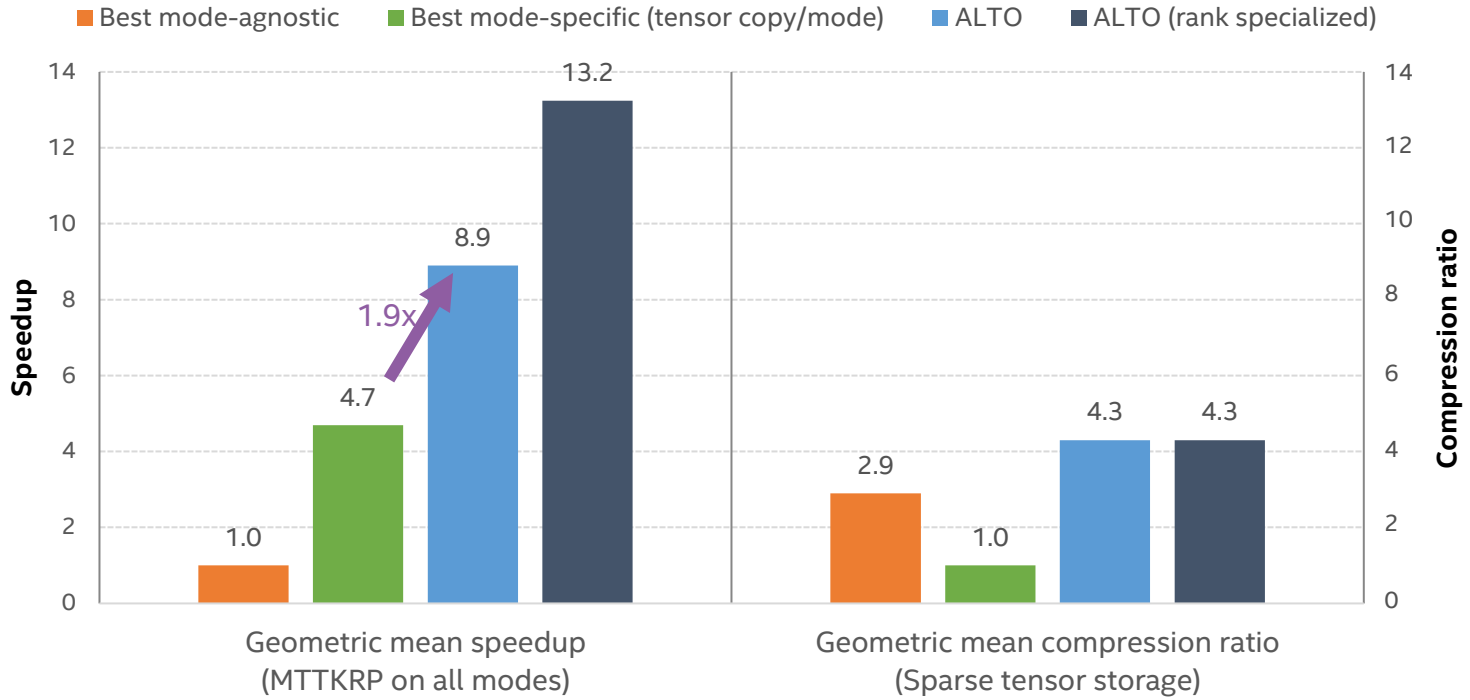

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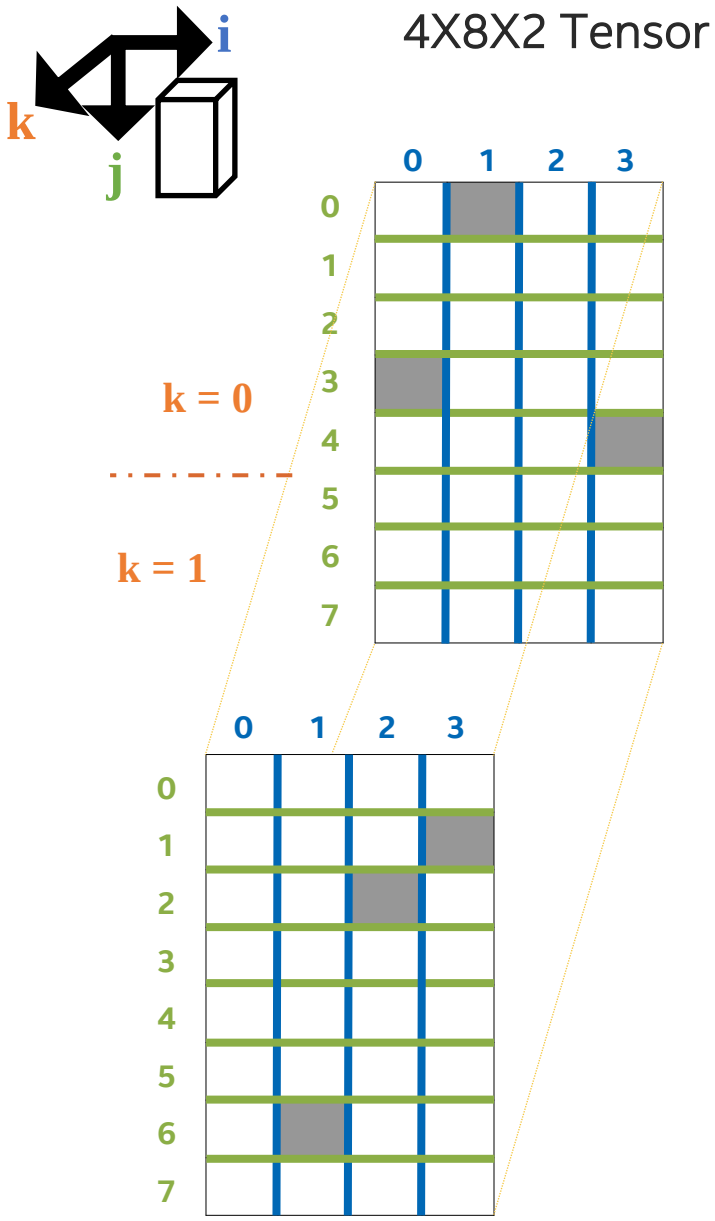

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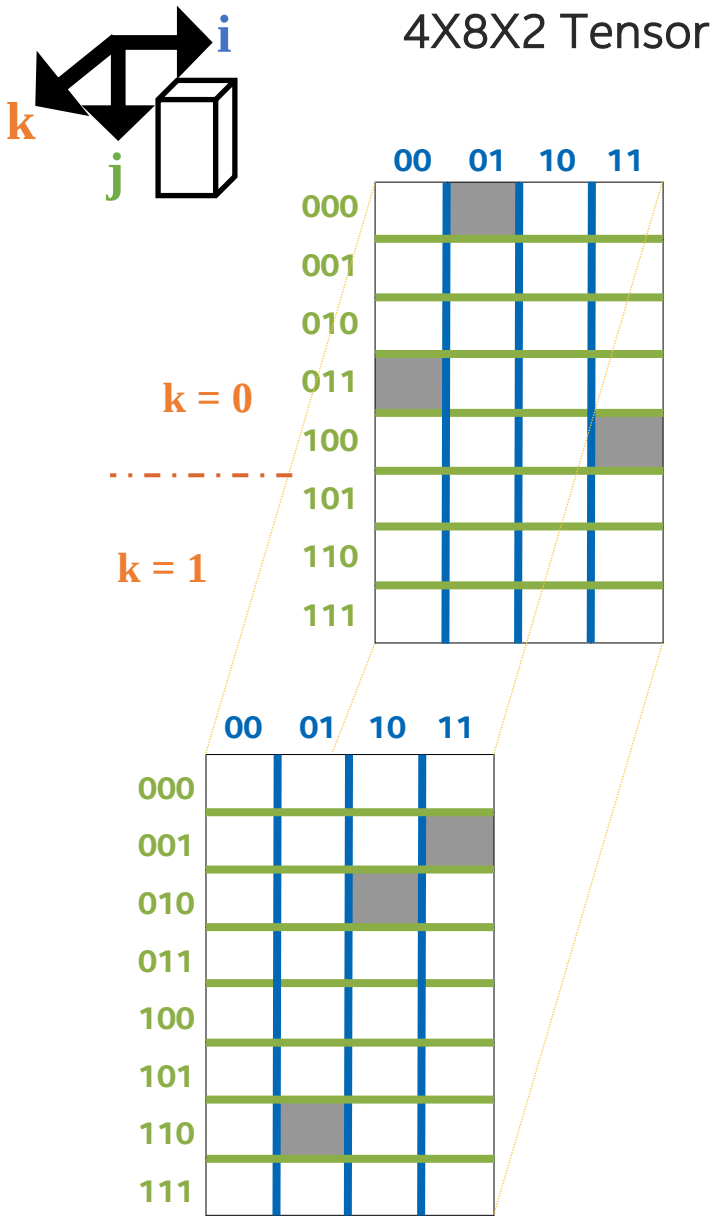

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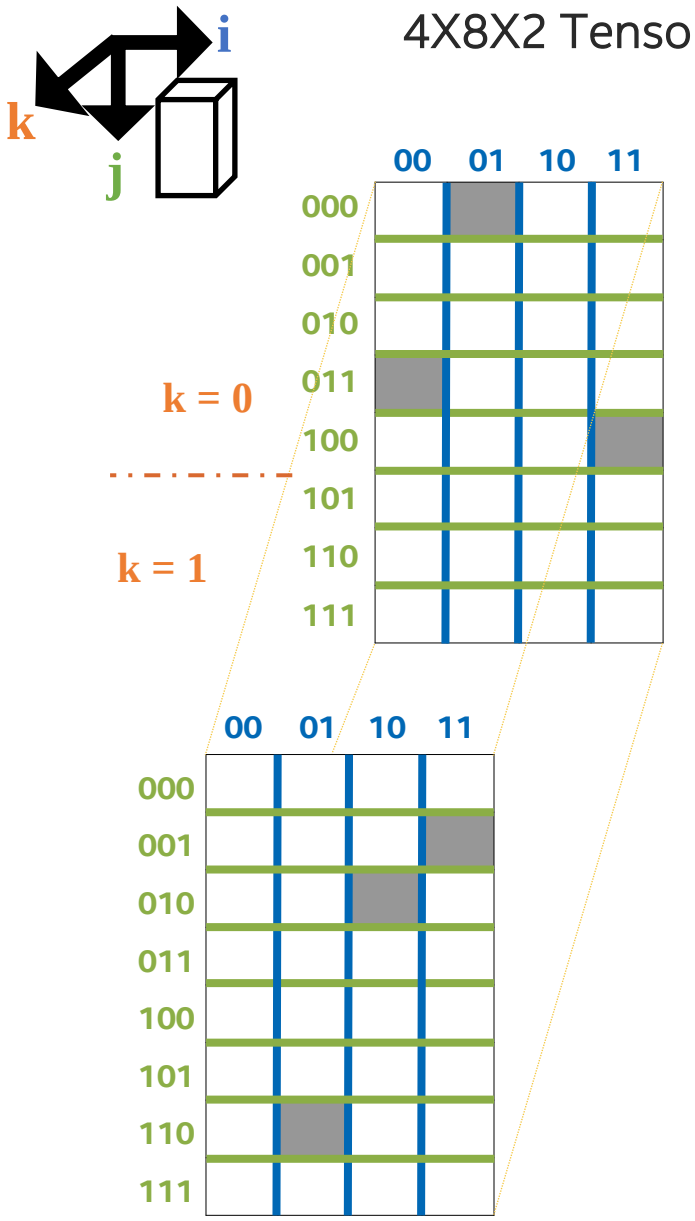

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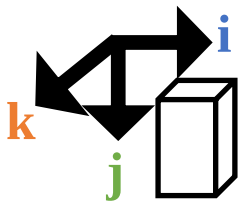
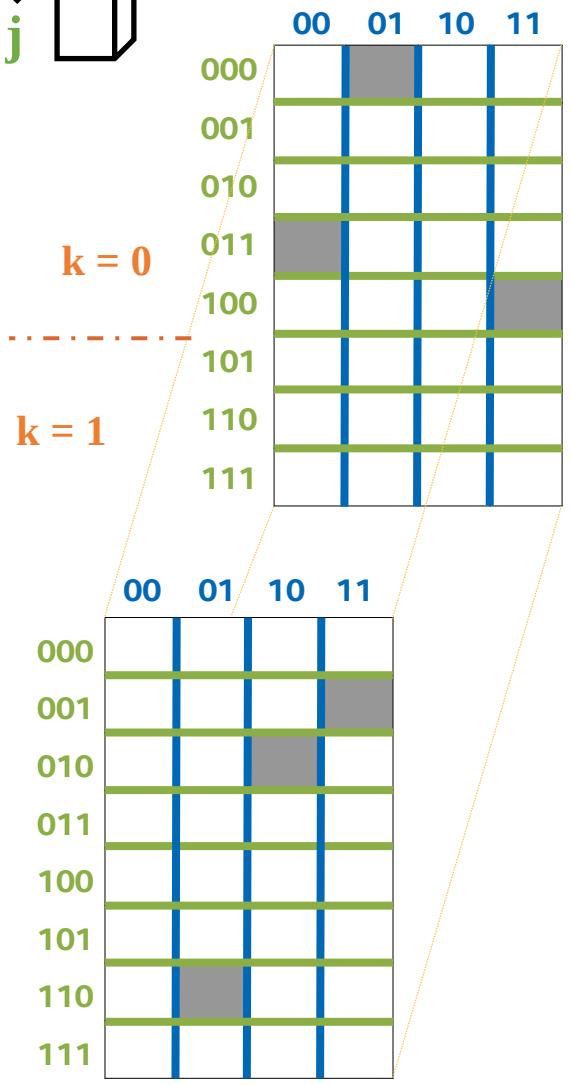
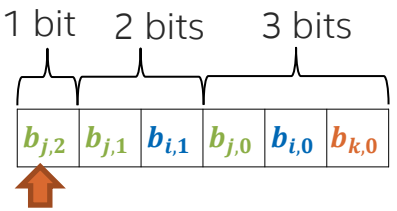
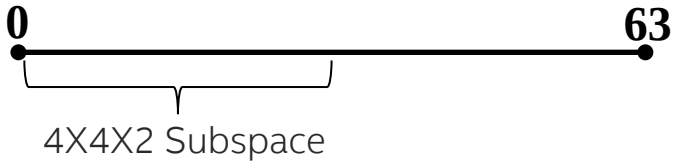


4X8X2 Tensor



ALTO Bit Mask





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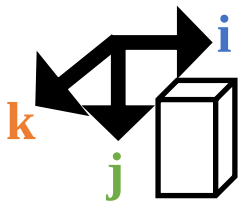

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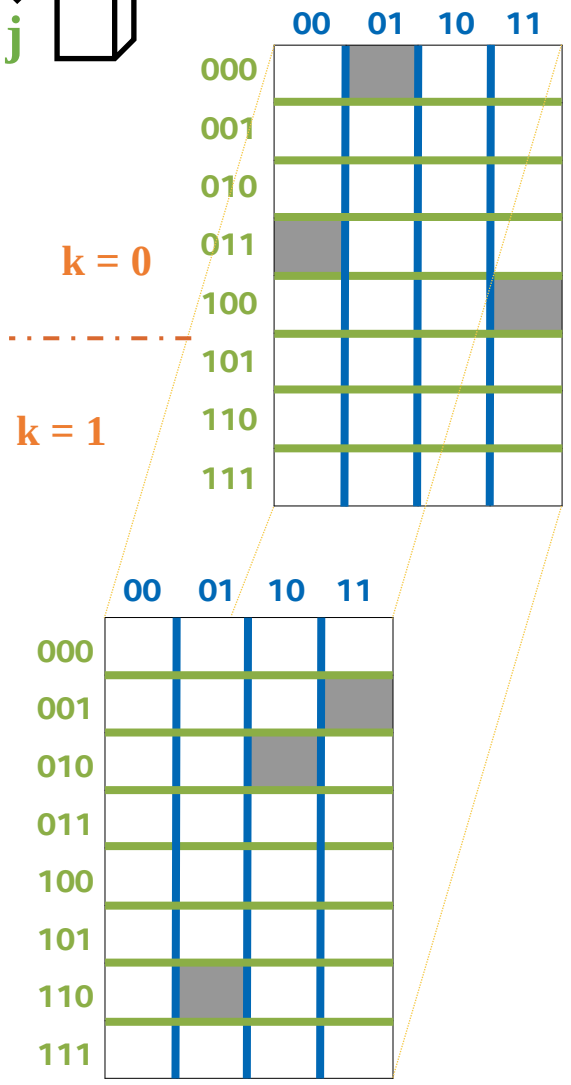

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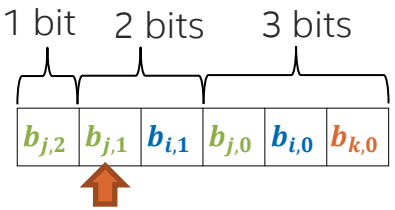

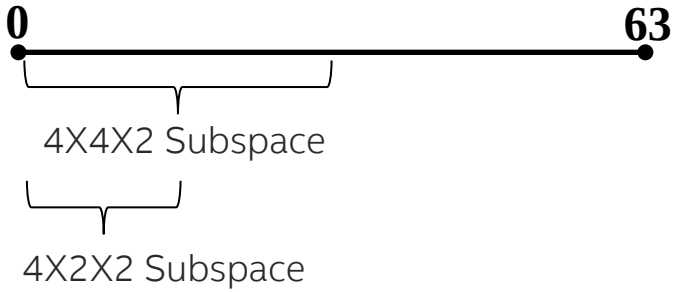

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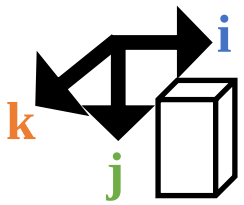

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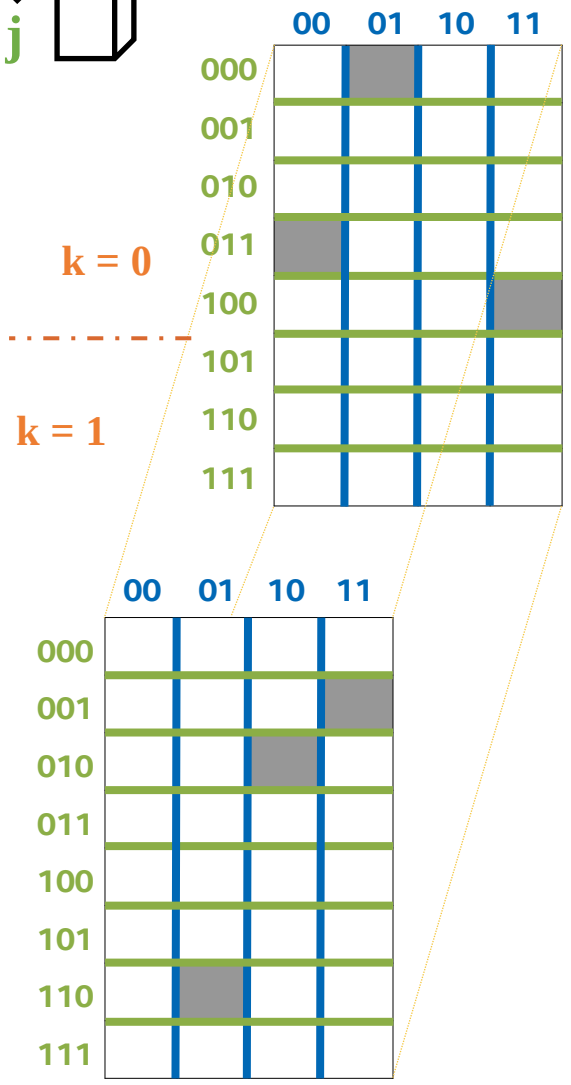

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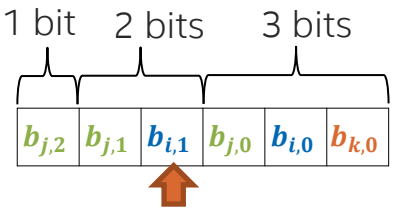

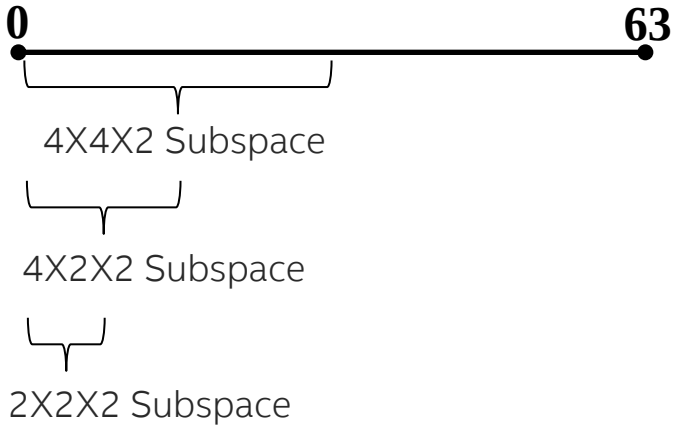

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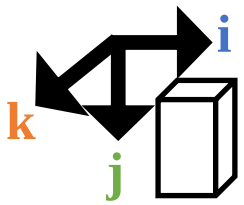

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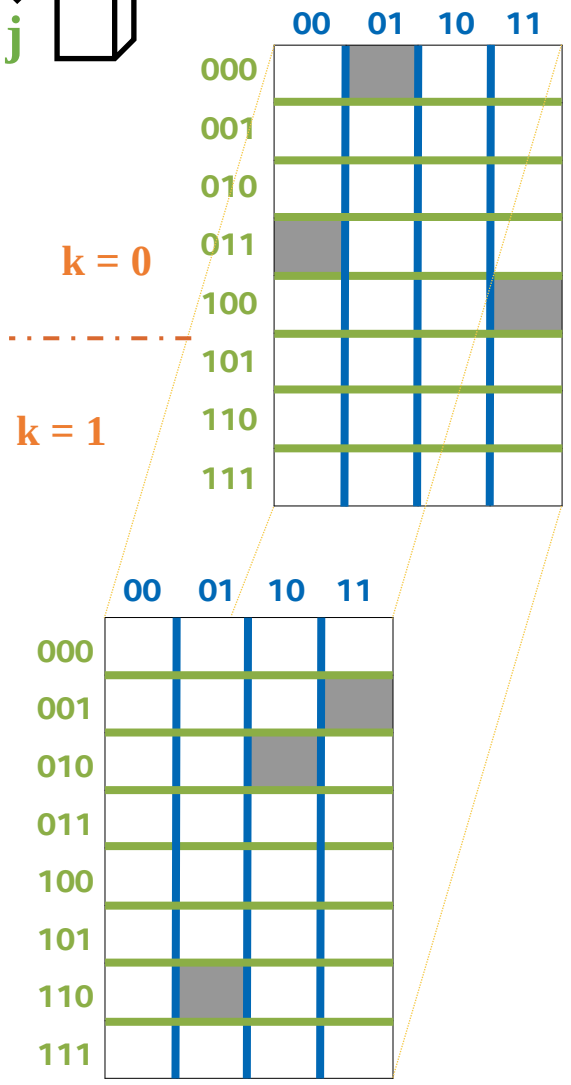

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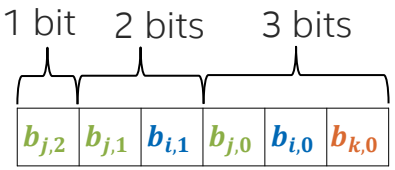

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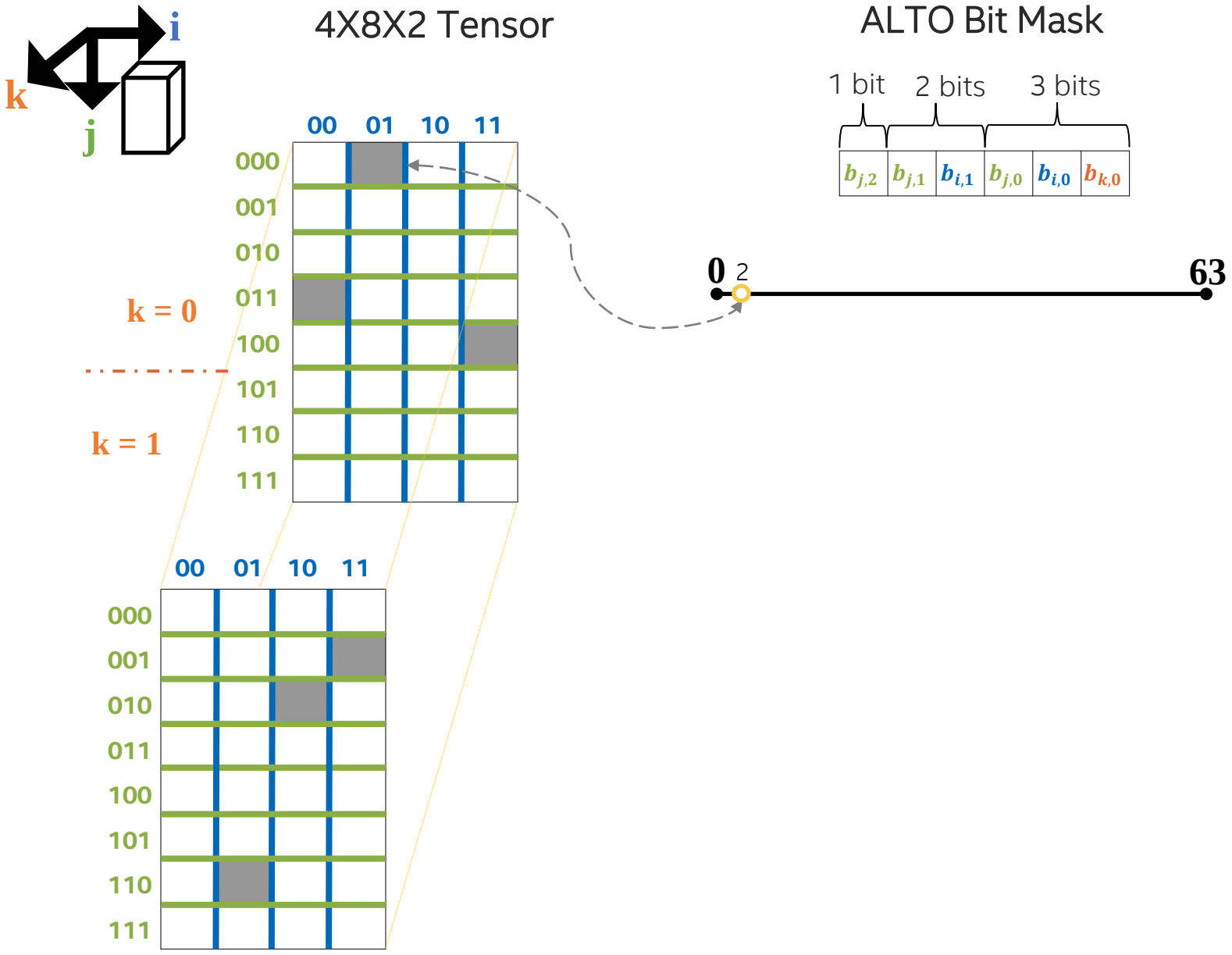

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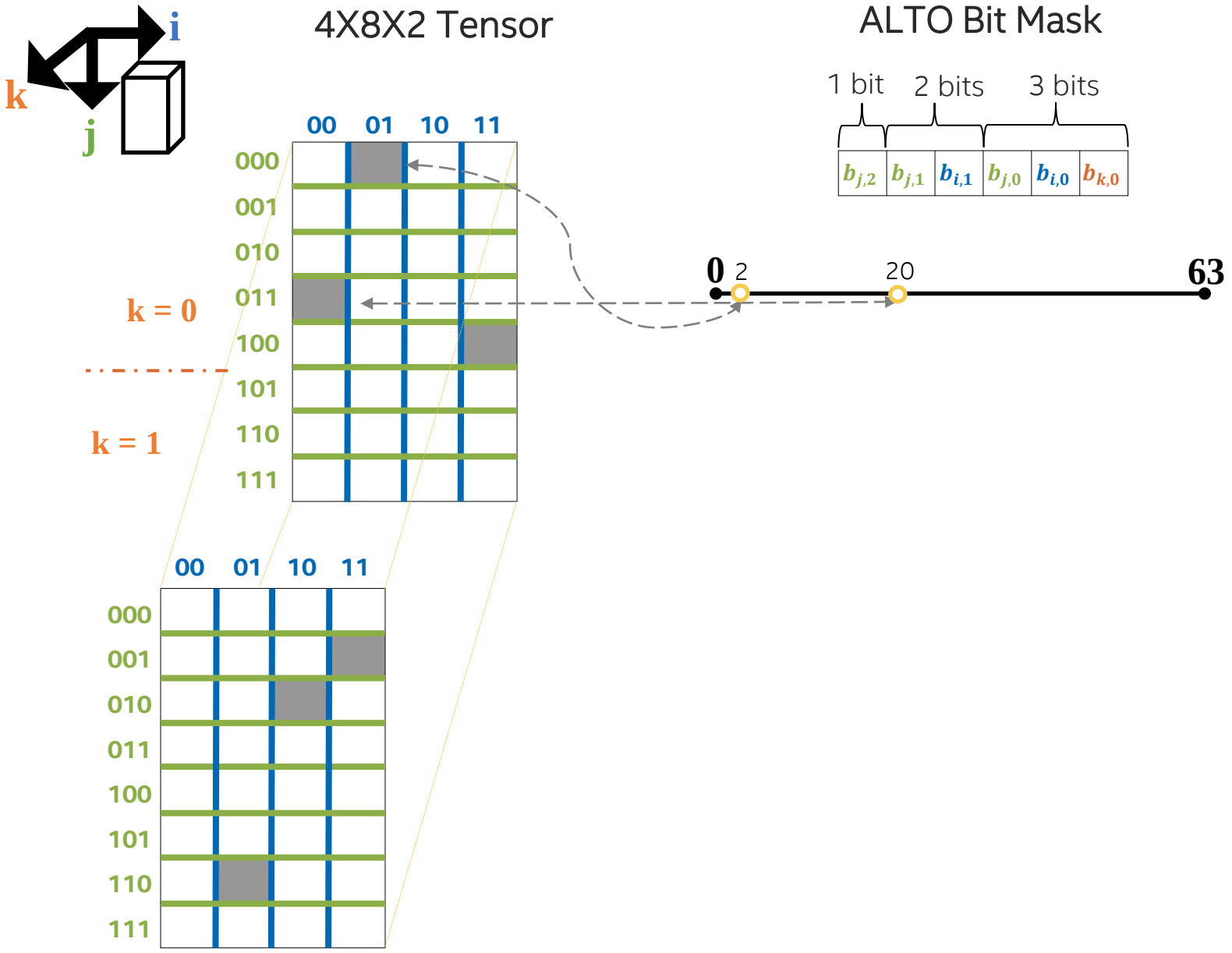

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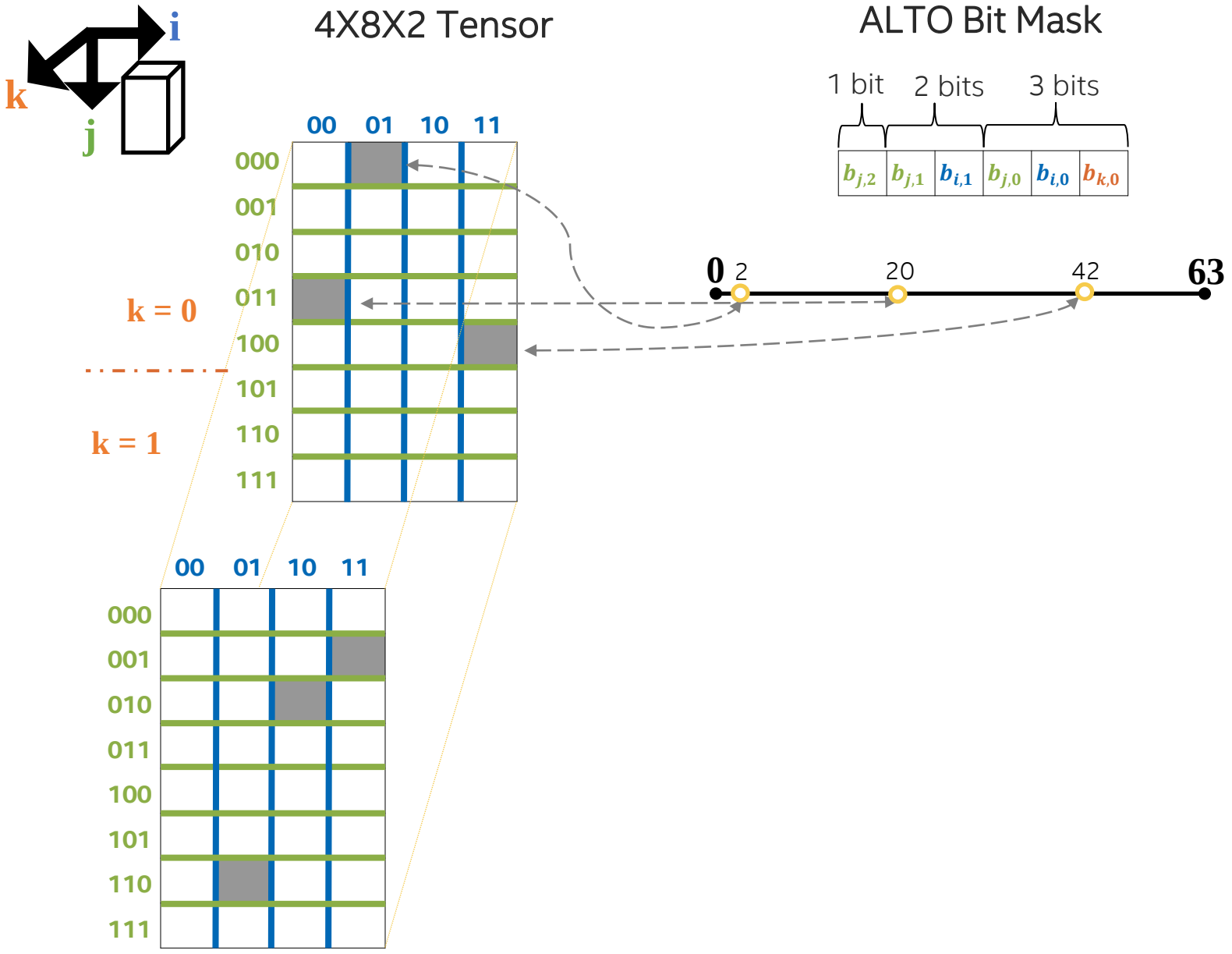

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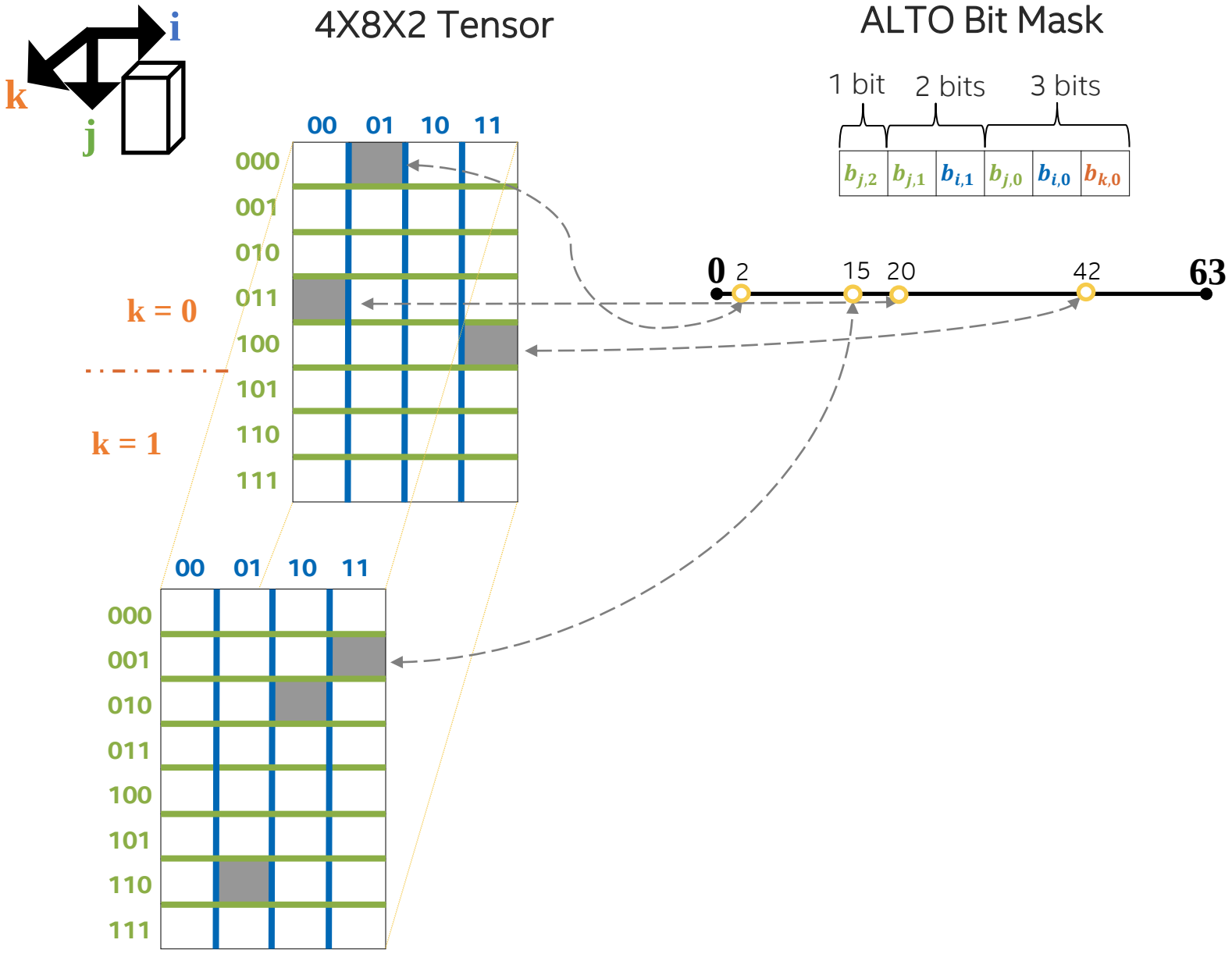

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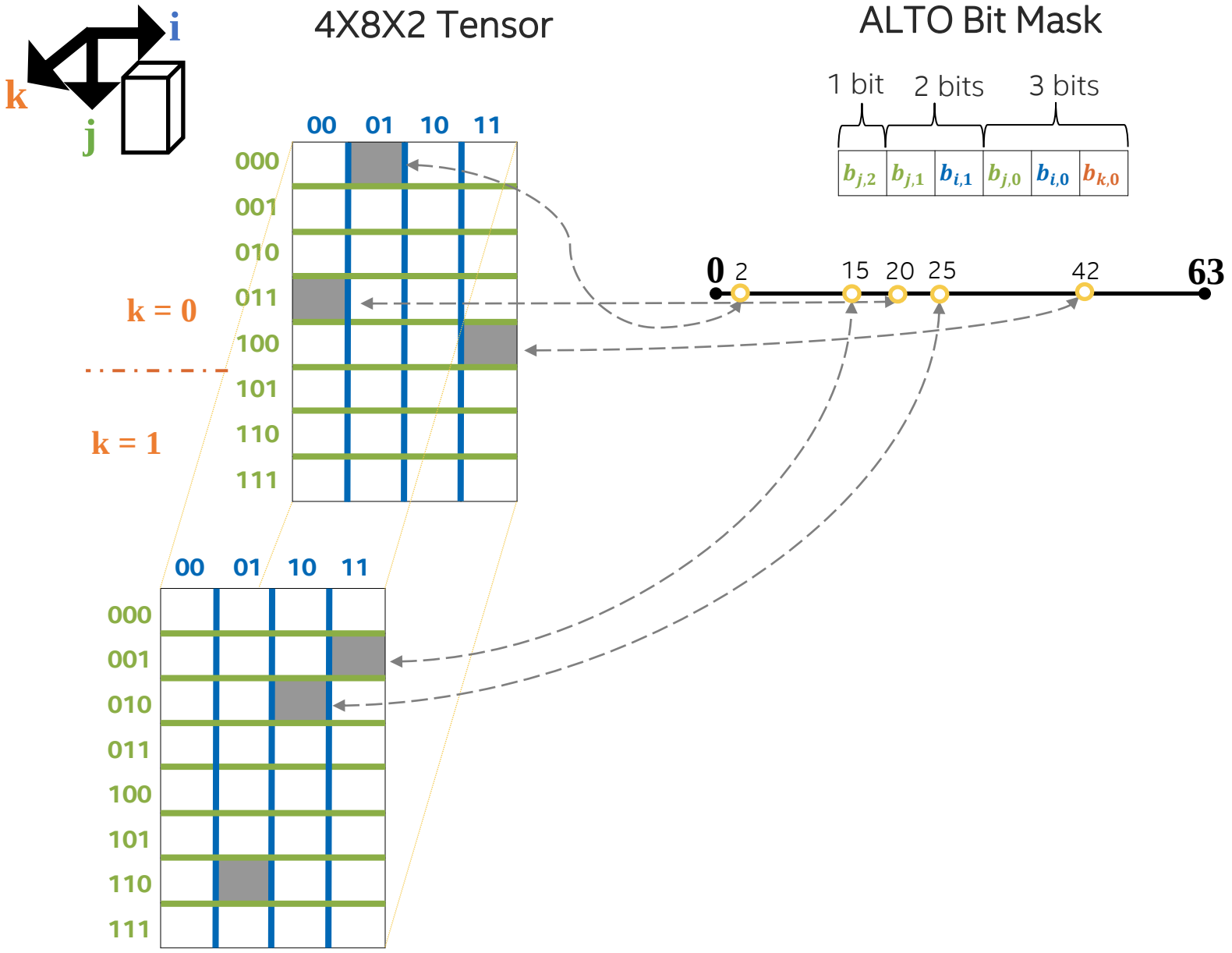

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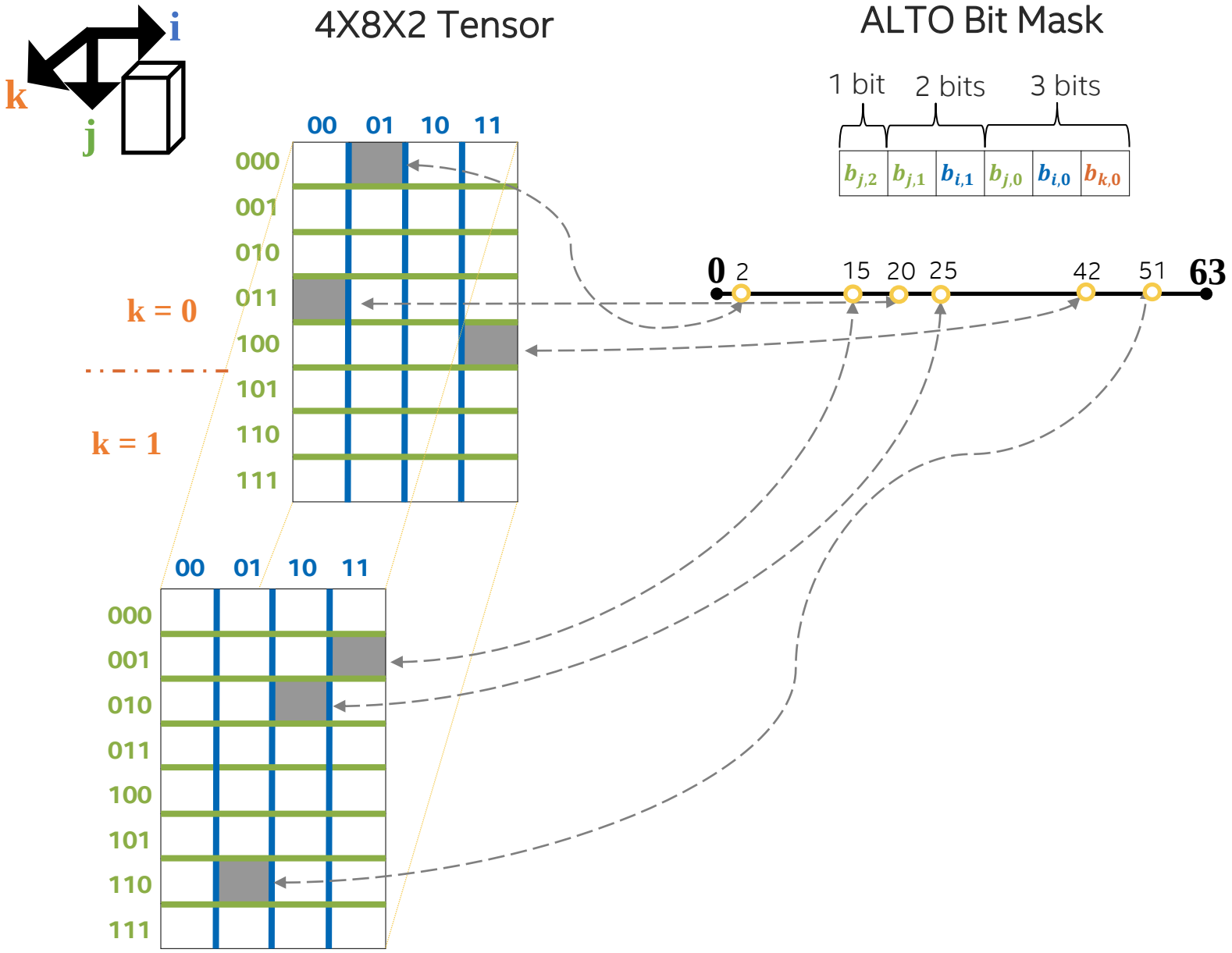

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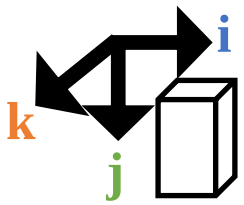

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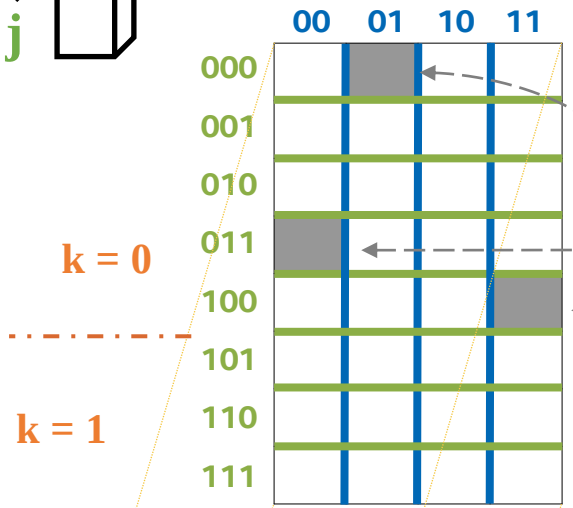

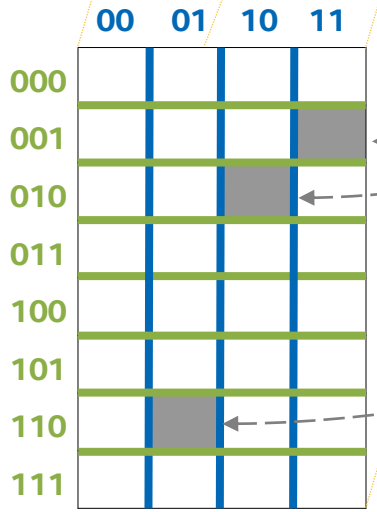

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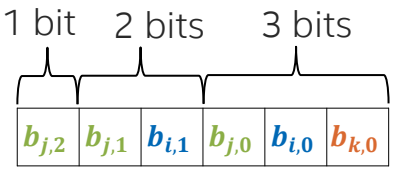
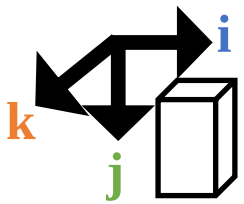


ALTO Tensor

Value	Position
$x_{1,0,0}$	2 (000010)
$x_{3,1,1}$	15 (001111)
$x_{0,3,0}$	20 (010100)
$x_{2,2,1}$	25 (011001)
$x_{3,4,0}$	42 (101010)
$x_{1,6,1}$	51 (110011)



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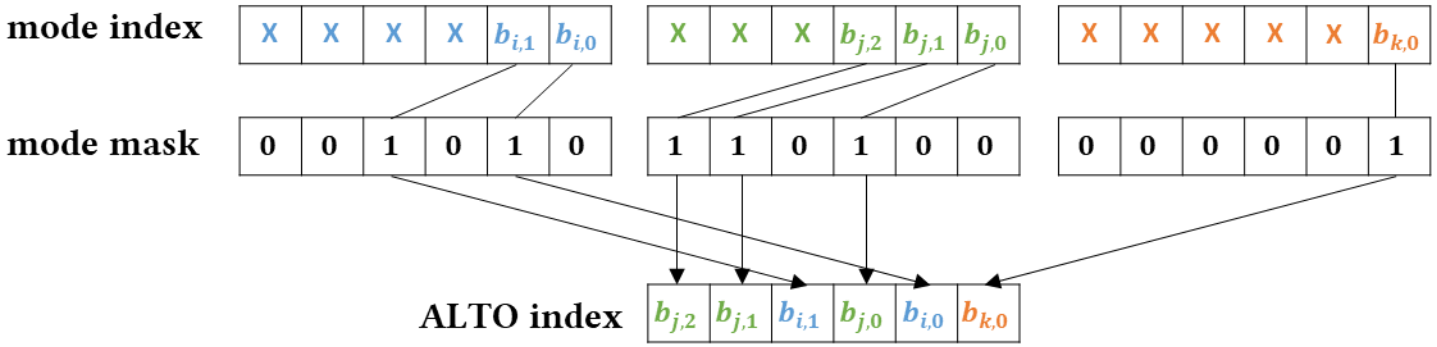
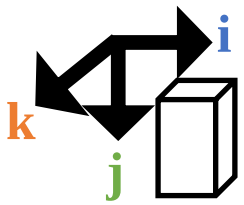


Encoding



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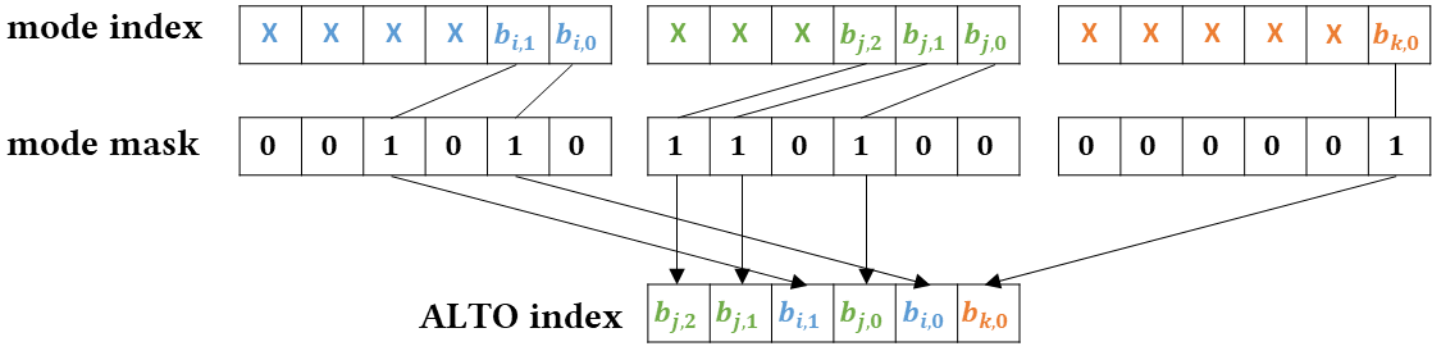

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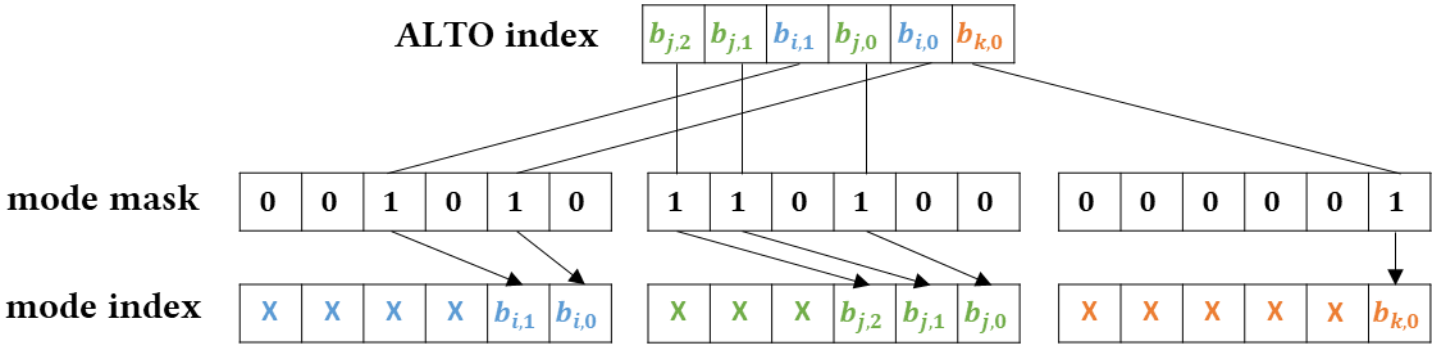
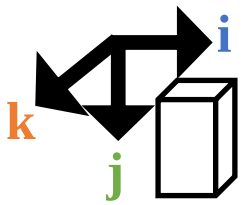
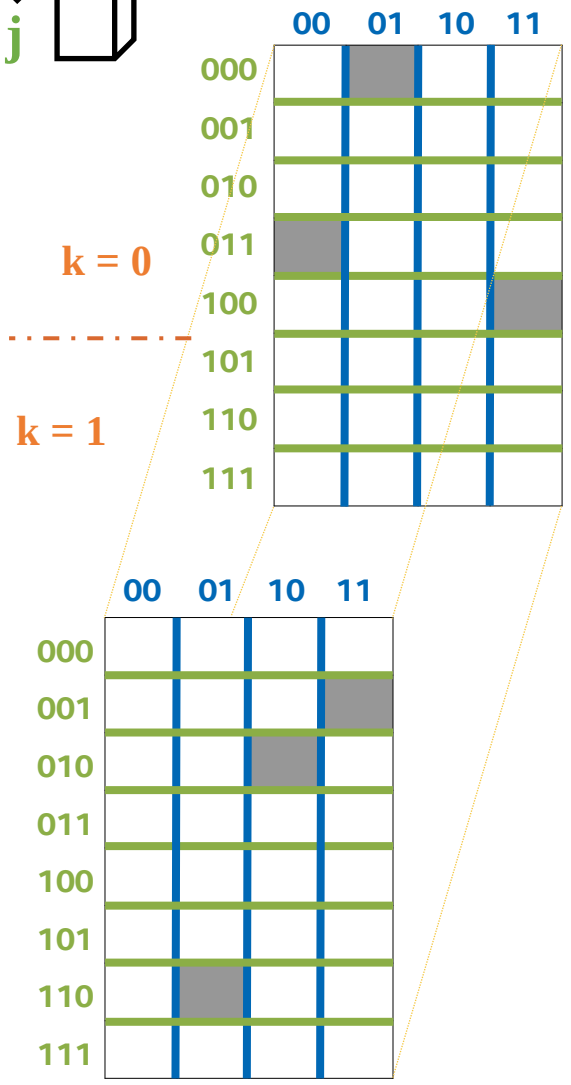


Decoding

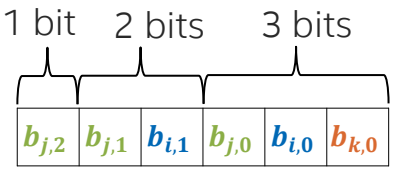
ALTO: Example



4X8X2 Tensor



ALTO Bit Mask



ALTO Tensor

Value	Position
$x_{1,0,0}$	2 (000010)
$x_{3,1,1}$	15 (001111)
$x_{0,3,0}$	20 (010100)
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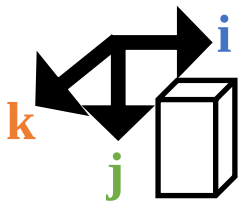
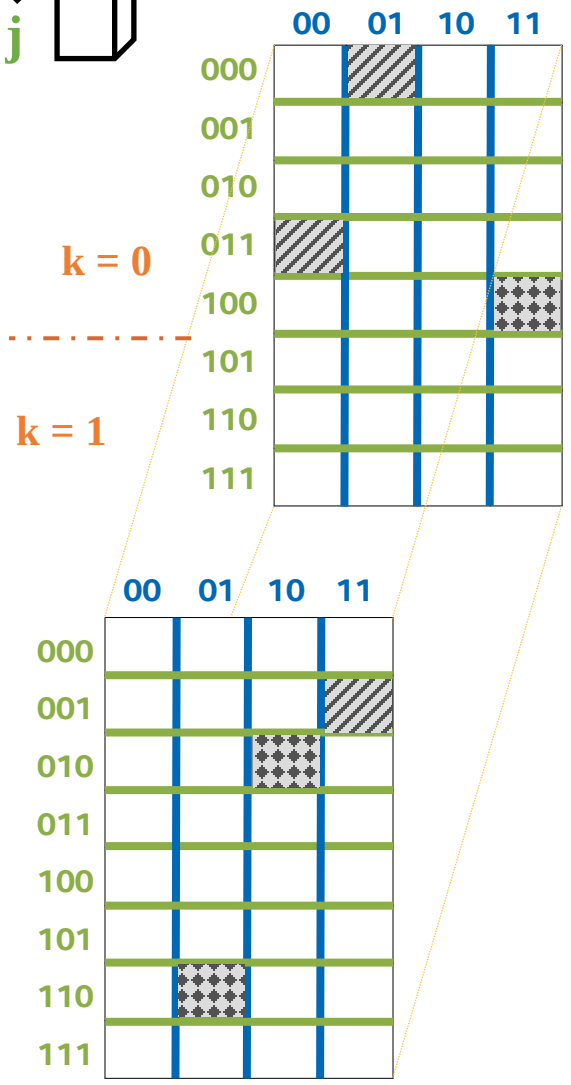
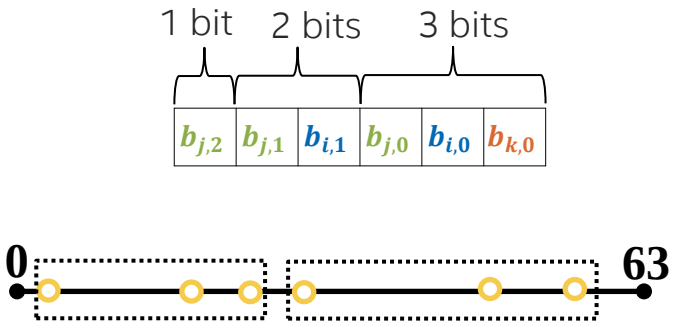
ALTO: Example



4X8X2 Tensor



ALTO Bit Mask



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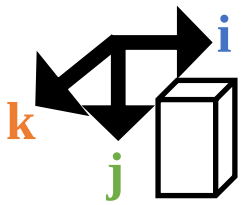

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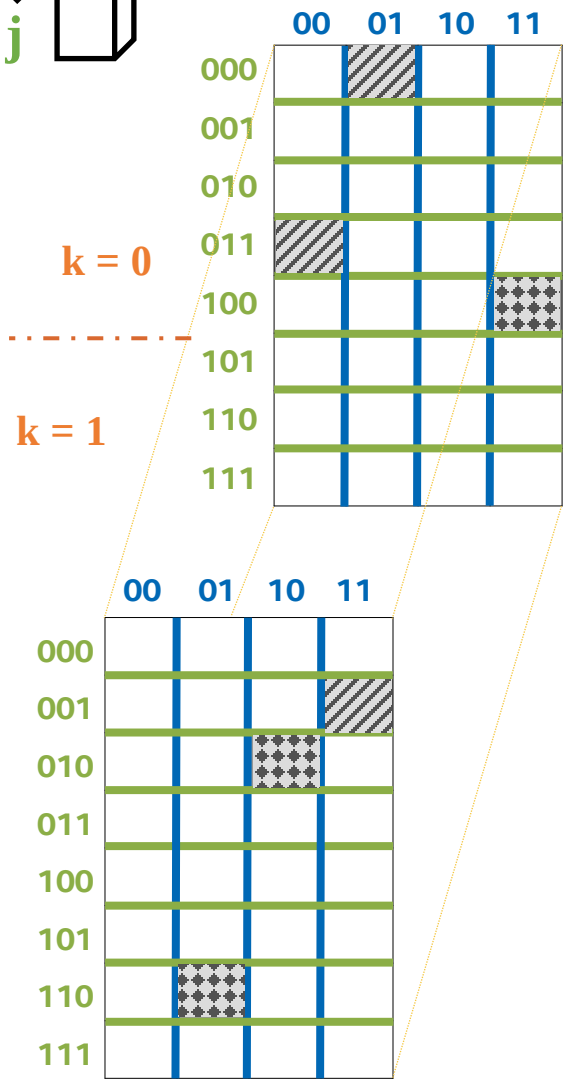

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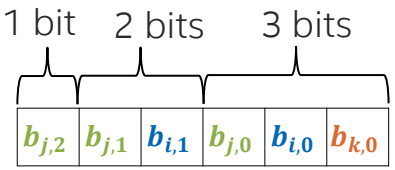

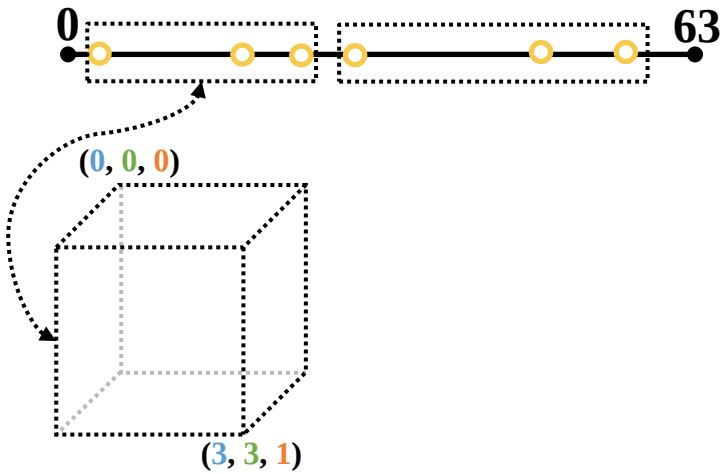

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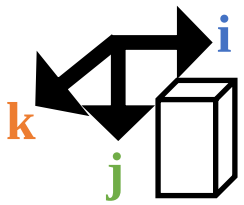

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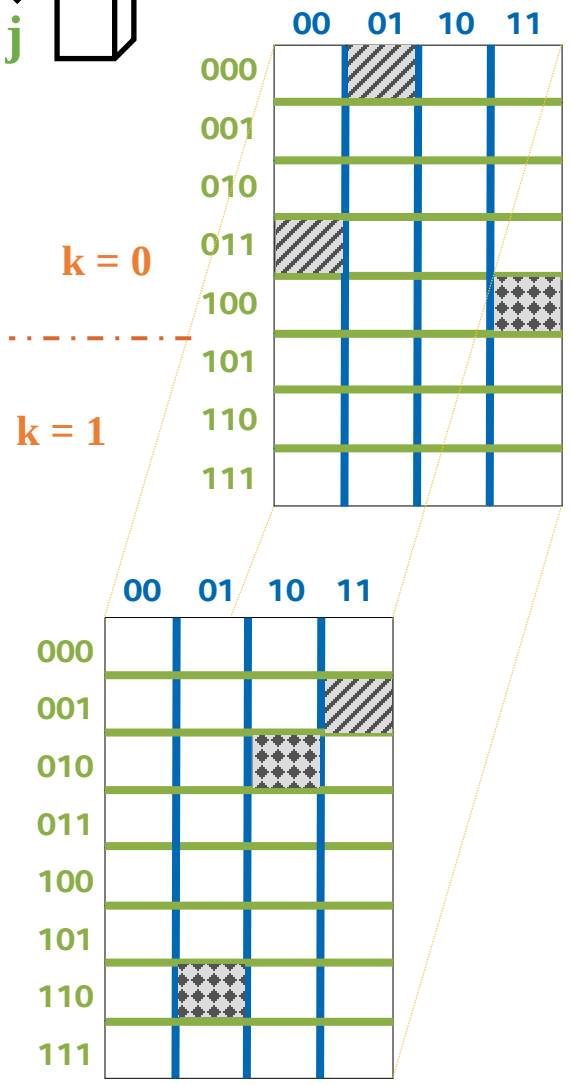

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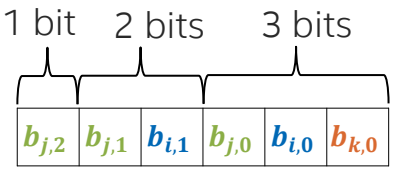

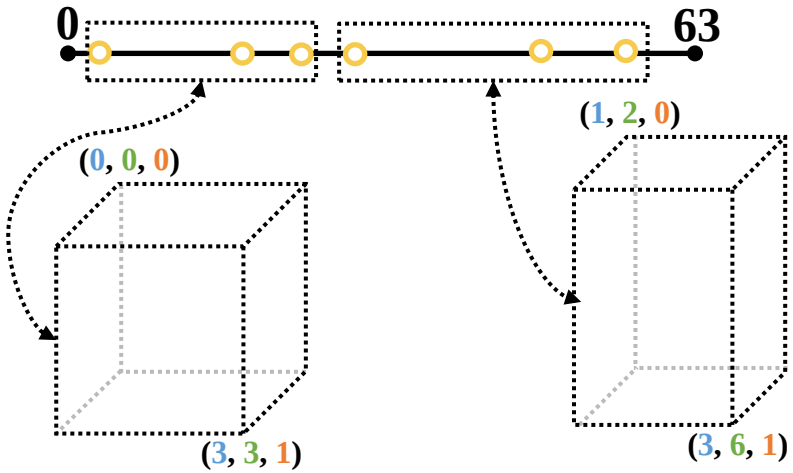

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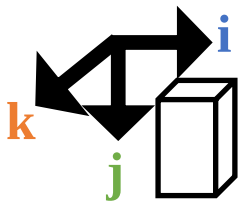

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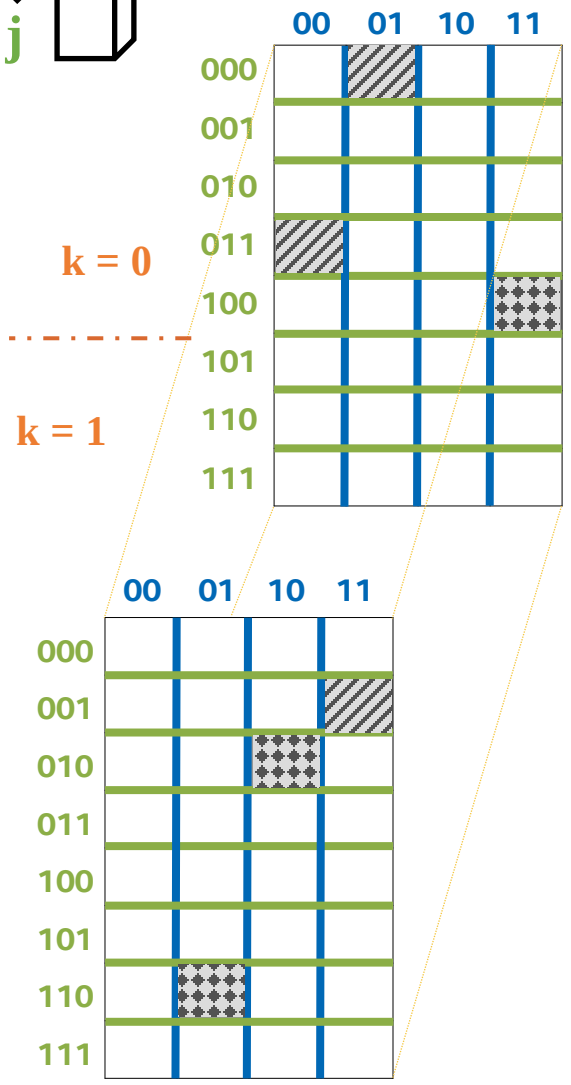

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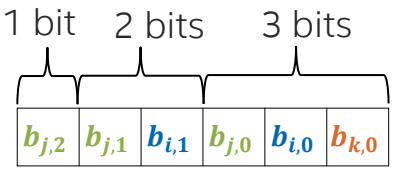

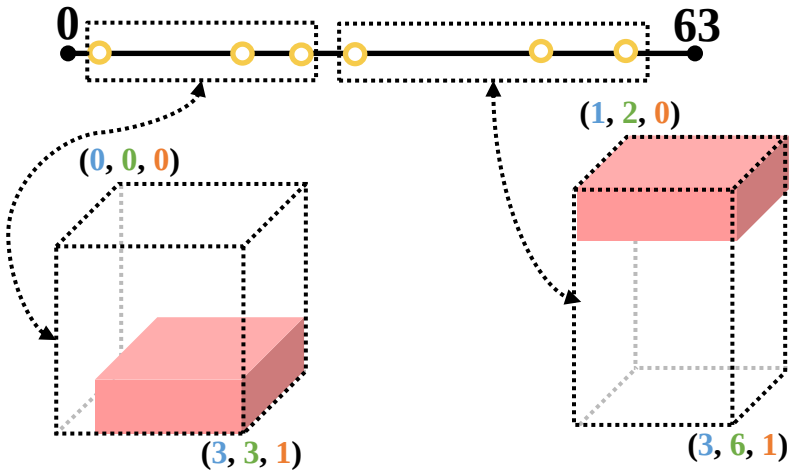

overlapping

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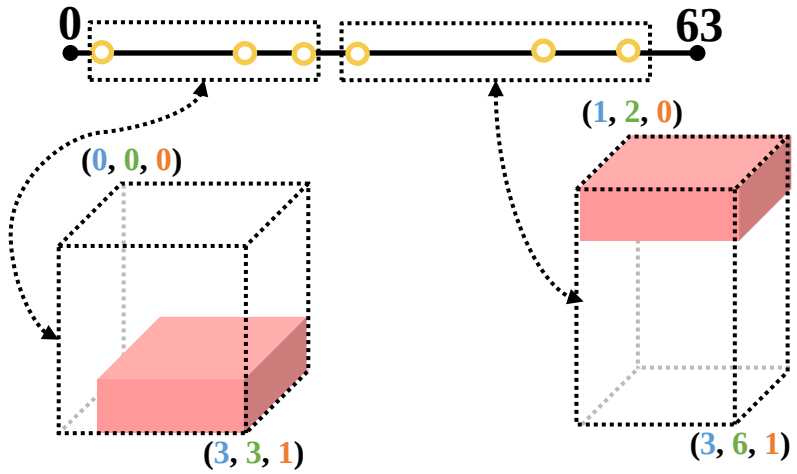
ALTO: Adaptive conflict resolution



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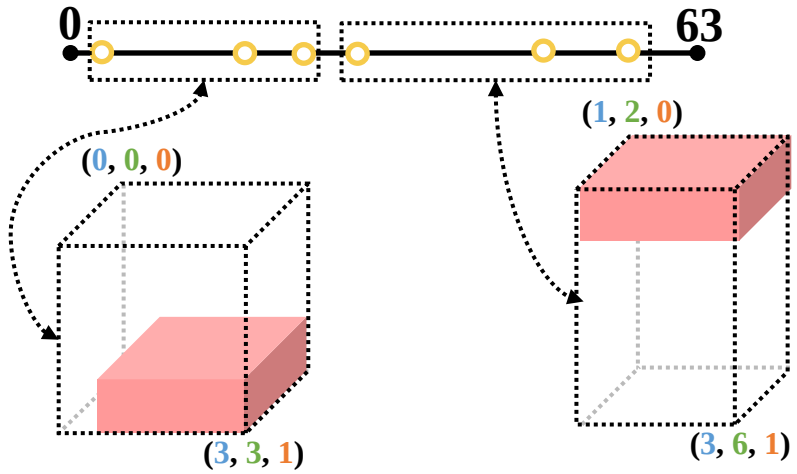

- Synchronization based on **data reuse** of target tensors

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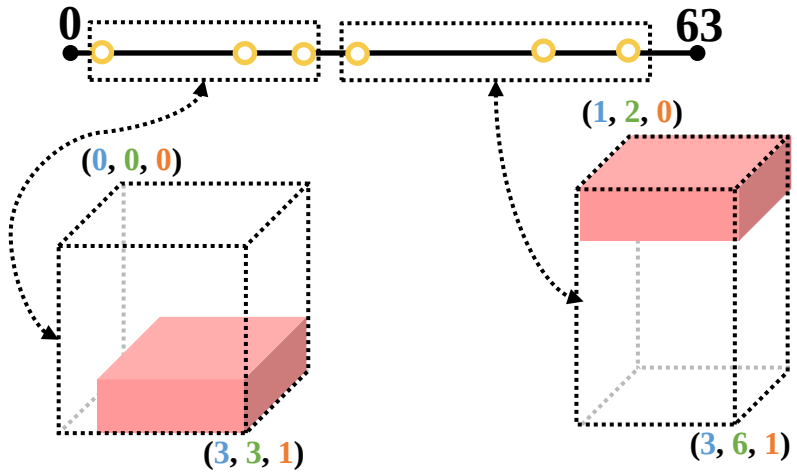

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  1. Limited or no data reuse ➔ conflict resolution via **atomic updates**

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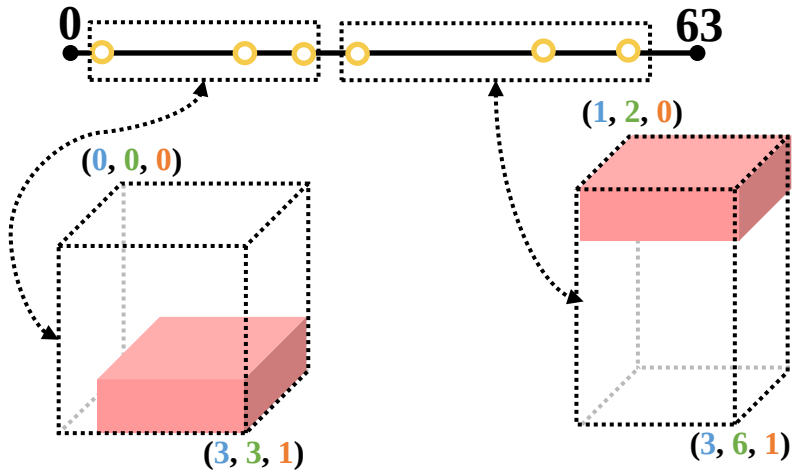

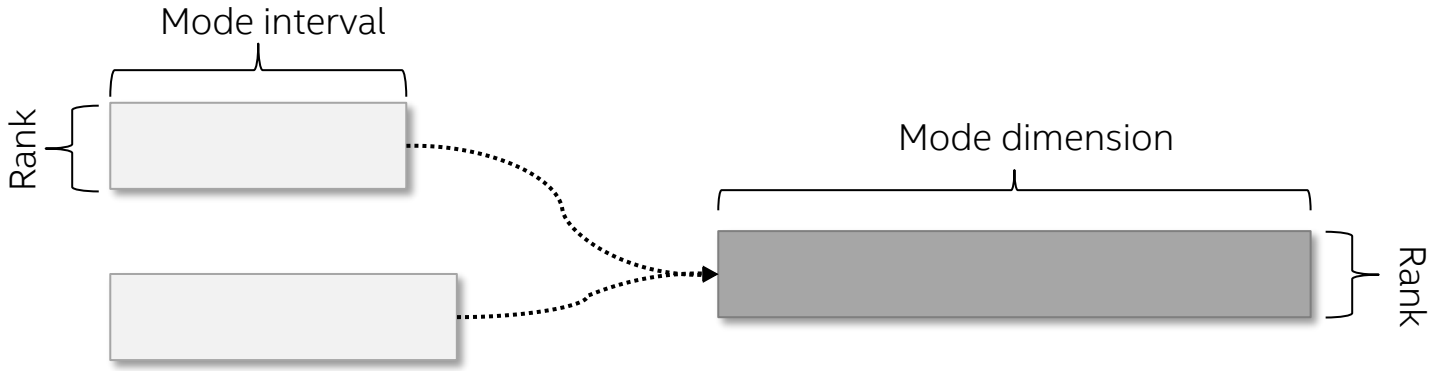

- Synchronization based on **data reuse** of target tensors
  1. Limited or no data reuse ➔ conflict resolution via **atomic updates**
  2. Otherwise ➔ **local temporary storage** and **global parallel reduction**

ALTO: Put it all together (MTTKRP)

Algorithm Adaptive parallel execution of mode-1 MTTKRP-ALTO kernel. ALTO automatically uses local storage or atomics, based on the reuse of output fibers, to resolve the update conflicts.

Input: A third-order ALTO sparse tensor $\mathcal{X} \in \mathbb{R}^{I \times J \times K}$ with M nonzero elements, dense factor matrices $\mathbf{A} \in \mathbb{R}^{I \times R}$, $\mathbf{B} \in \mathbb{R}^{J \times R}$, and $\mathbf{C} \in \mathbb{R}^{K \times R}$

Output: Updated dense factor matrix $\tilde{\mathbf{A}} \in \mathbb{R}^{I \times R}$

1: **for** $l = 1, \ldots, L$ **in parallel do** ▷ ALTO line segments.
2: **for** $\forall x \in \mathcal{X}_l$ **do**
3: $i = \text{EXTRACT}(pos(x), MASK(1))$ ▷ De-linearization.
4: $j = \text{EXTRACT}(pos(x), MASK(2))$
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6: **for** $r = 1, \ldots, R$ **do**
7: $\mathbf{Temp}_l(i - T^s_{l,1}, r) \mathrel{+}= val(x) \times \mathbf{B}(j, r) \times \mathbf{C}(k, r)$
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11: **end for**
12: **for** $b = 1, \ldots, I$ **in parallel do** ▷ Pull-based accumulation.
13: **for** $\forall l$ where $b \in [T^s_{l,1}, T^e_{l,1}]$ **do**
14: **for** $r = 1, \ldots, R$ **do**
15: $\tilde{\mathbf{A}}(b, r) \mathrel{+}= \mathbf{Temp}_l(b - T^s_{l,1}, r)$
16: **end for**
17: **end for**
18: **end for**
19: **return** $\tilde{\mathbf{A}}$

Results

Platform

- Cascade Lake Platinum 8280
  - 2 sockets, 28 cores each, fixed @1.8 GHz

- Intel compiler and runtime
  - “-O3 -xCORE-AVX512 -qopt-zmm-usage=high” flags to fully utilize vector units
  - Parallel runs utilize all hardware threads (112) with thread pinning via LIKWID

Results

Configurations

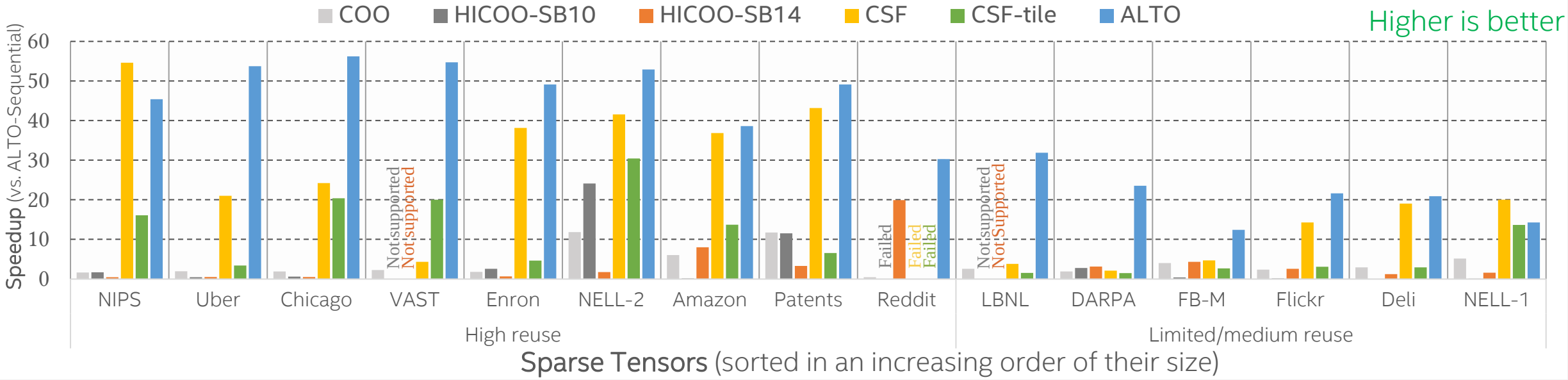
- MTTKRP with double-precision and native word (64-bit) data types
- Number of iterations = 100
- 15 tensors from FROSTT and HaTen2 datasets
  - NNZ range: [3 M, 4.7 B], Density range: [$1.5E^{-02}$, $4.3E^{-15}$]
- We consider the state-of-the-art sparse tensor libraries for CPUs
  - ParTI: coordinate (COO) and hierarchical COO (HiCOO) formats
  - SPLATT: compressed sparse fiber (CSF) and tiled CSF (CSF-tile)

Results

Parallel performance (all-modes MTTKRP)

- **47x** geomean speedup for high reuse cases (**84%** of realizable theoretical speedup)
- **22x** geomean speedup for memory-bound cases

- Prior formats' performance varies widely due to sensitivity to irregularities in shape and data



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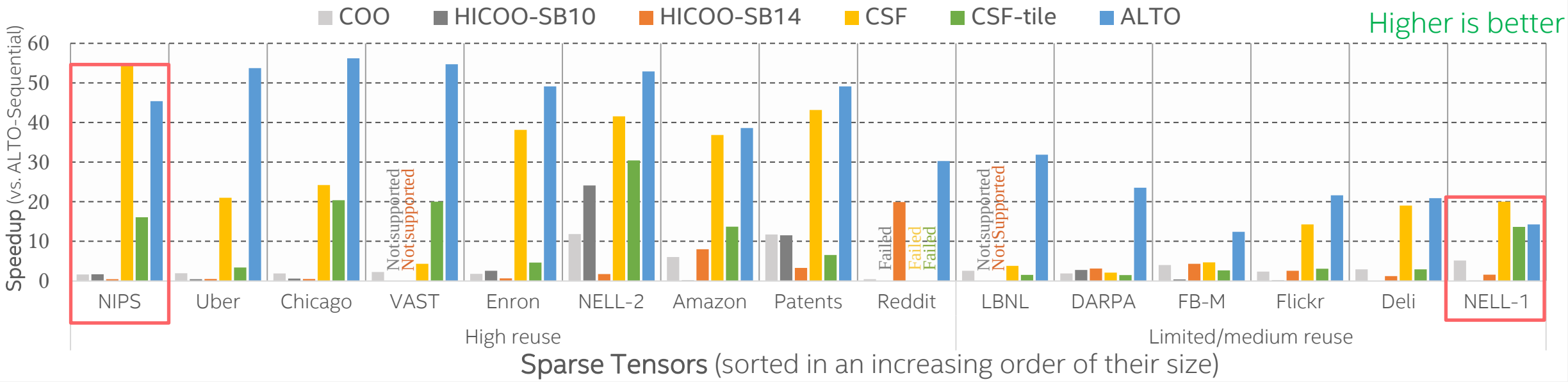
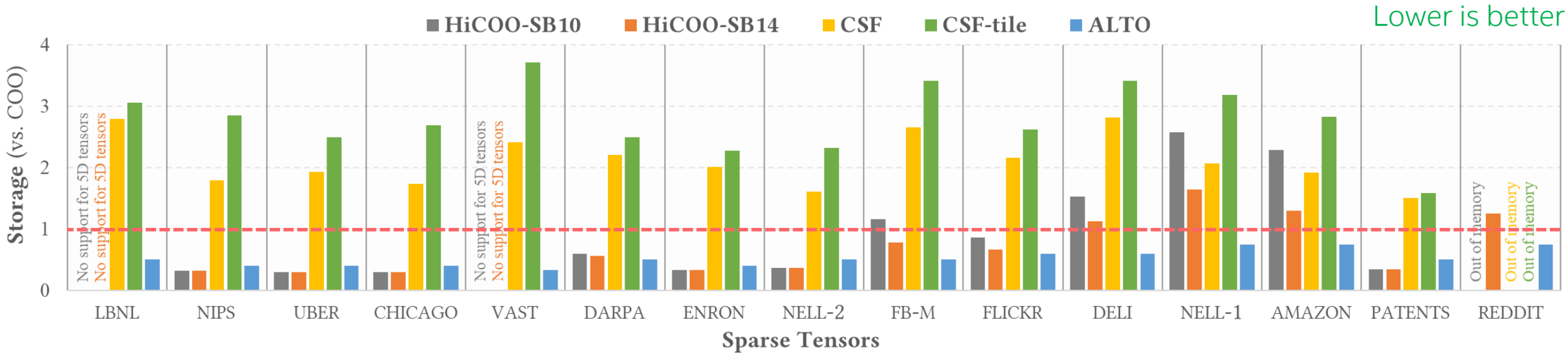


Results

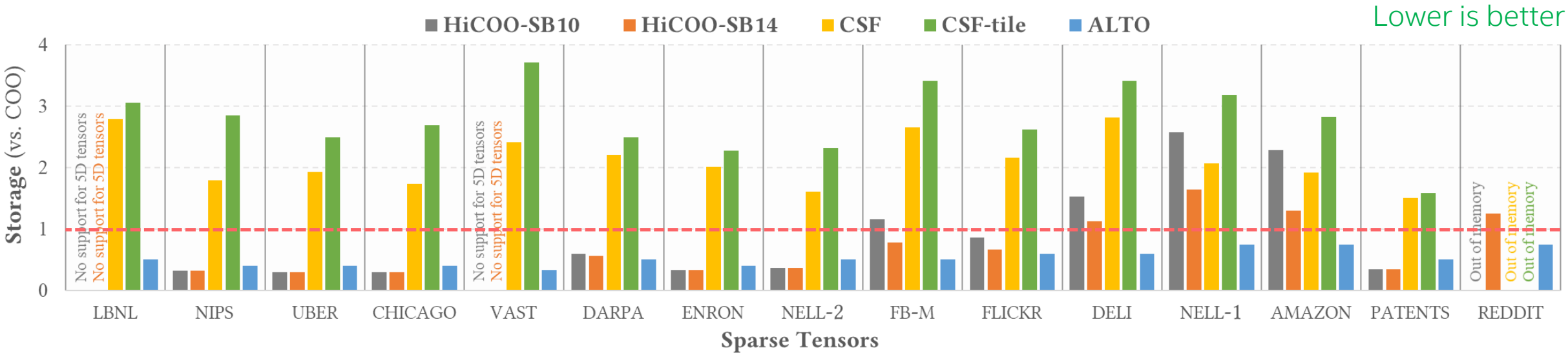
Tensor storage



Results

Tensor storage

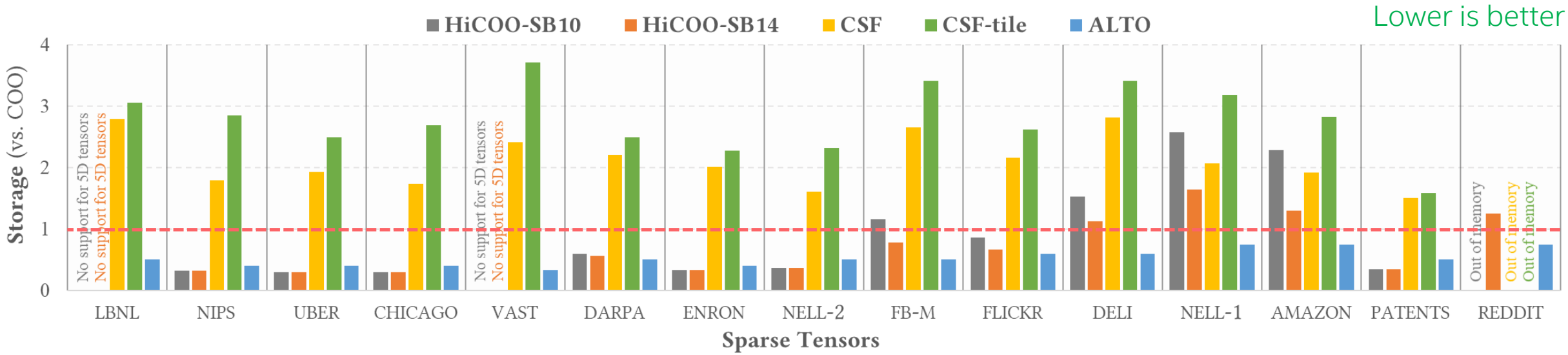
- ALTO format storage < COO



Results

Tensor storage

- ALTO format storage < COO
- Mode-specific formats' storage > COO
- imposing a tilling over CSF increases storage requirements



Results

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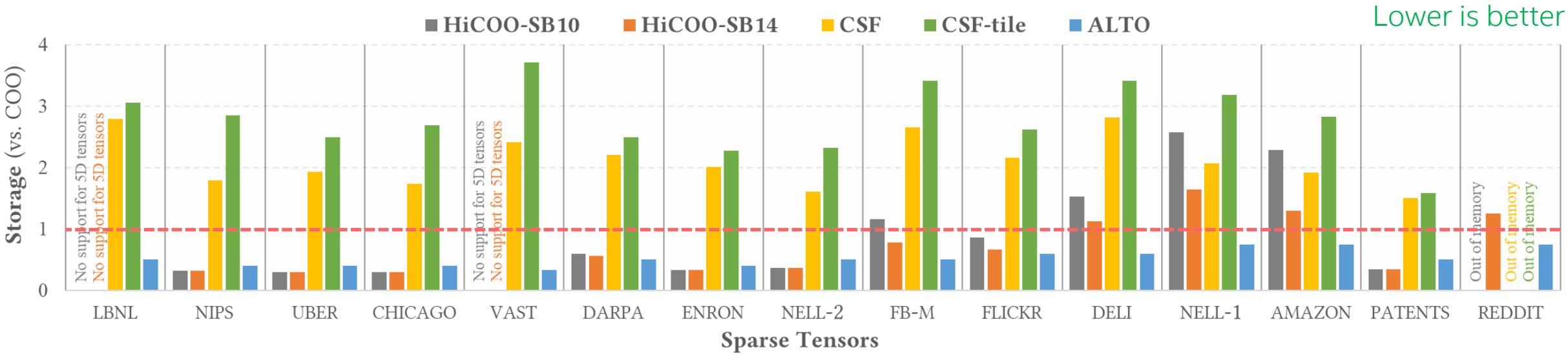
- HiCOO's memory consumption depending on
  a) block/superblock sizes
  b) spatial distribution of nonzero elements



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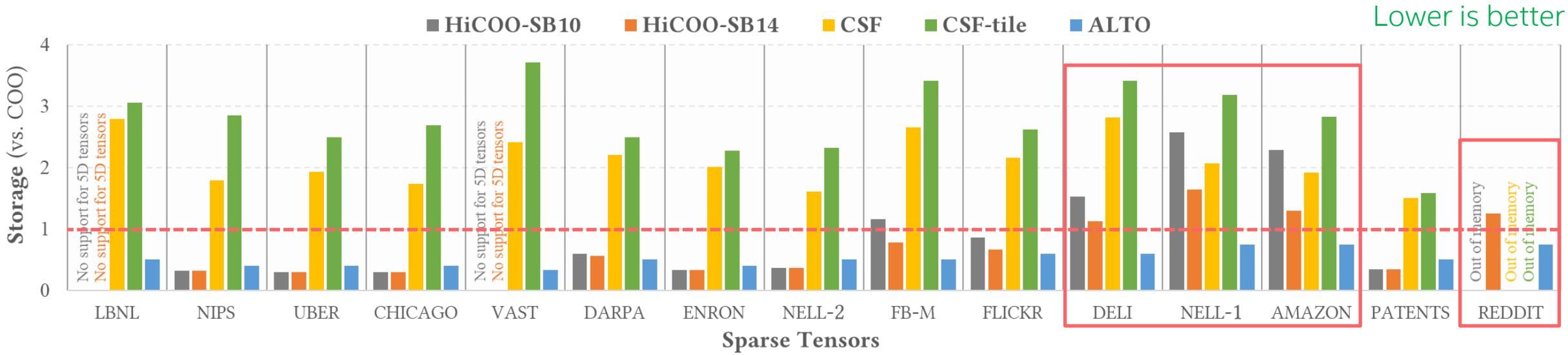

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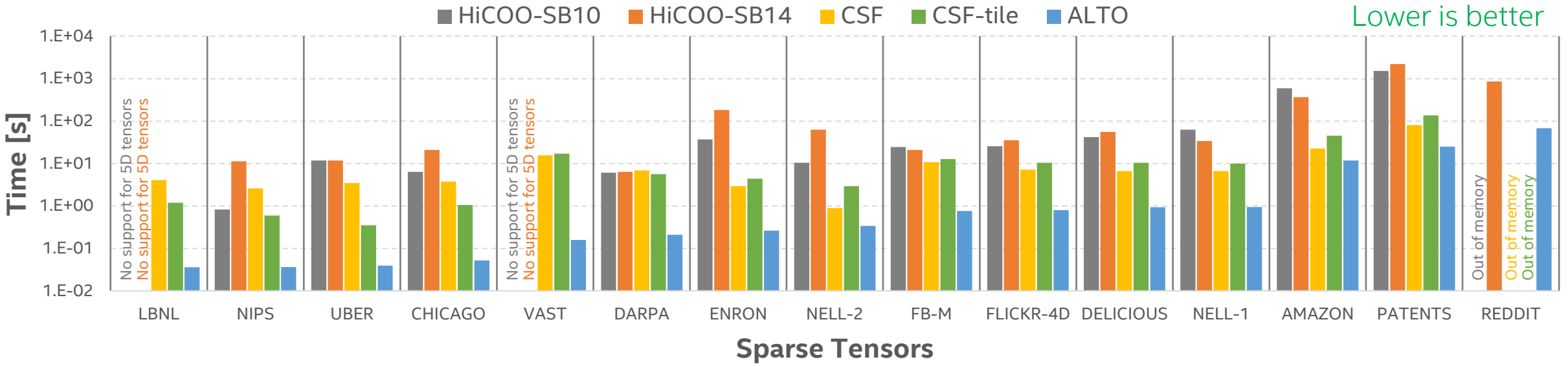
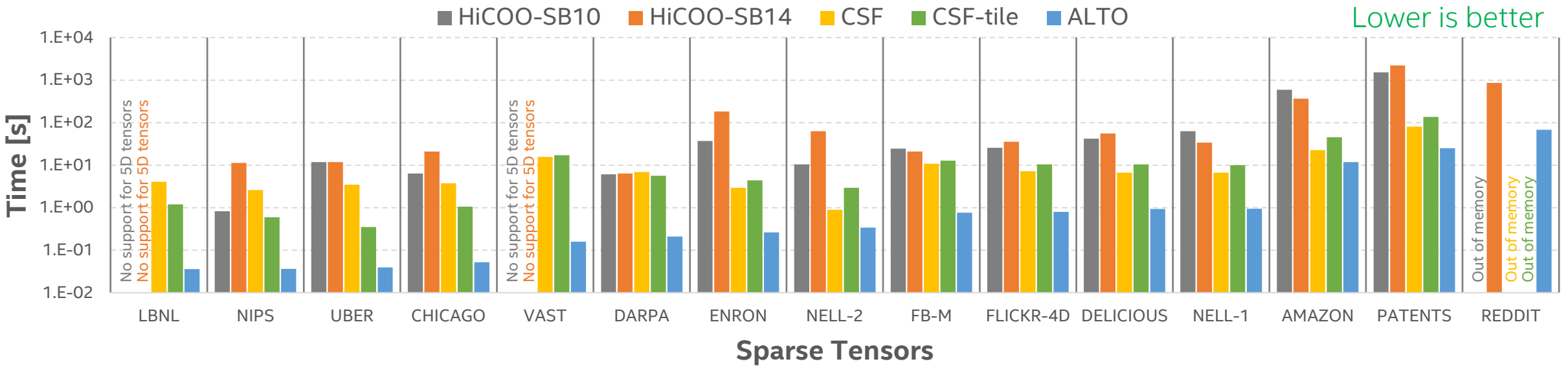
ALTO generation cost



Results

ALTO generation cost

- ALTO substantially **decreases** sorting costs



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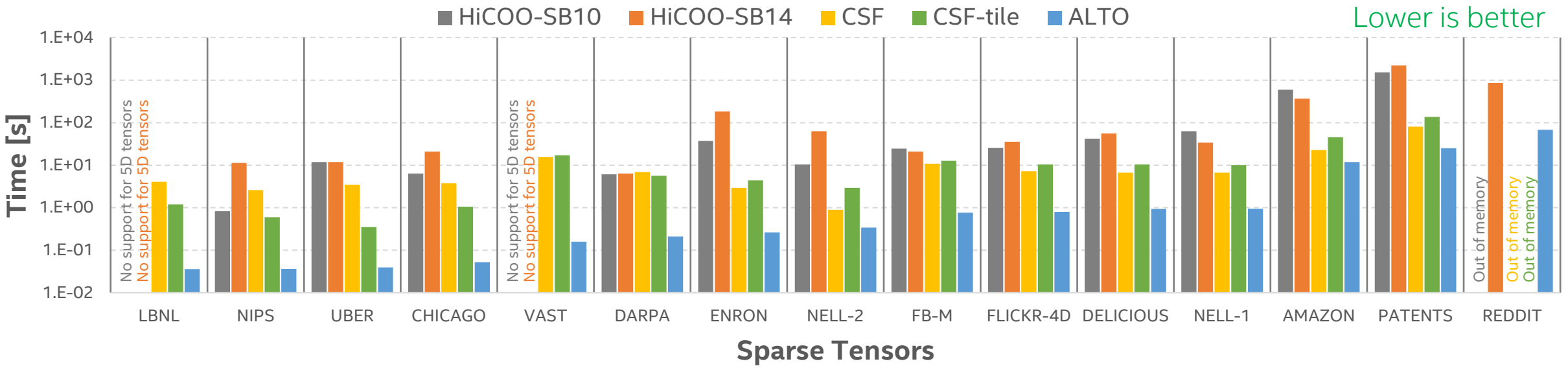
ALTO generation cost

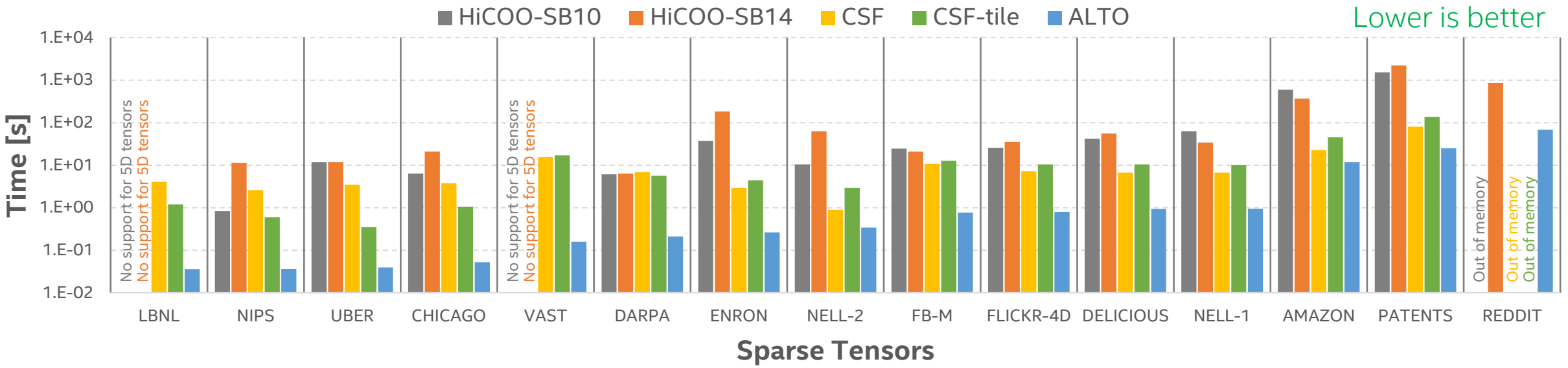
- ALTO substantially **decreases** sorting costs
- Block-based formats require expensive clustering & scheduling of NNZs



Results

ALTO generation cost

- ALTO substantially **decreases** sorting costs
- Block-based formats require expensive clustering & scheduling of NNZs
- CSF formats generated from ***presorted*** tensors; still slower than ALTO



Conclusion

- The ALTO format ***efficiently encodes*** sparse tensors of arbitrary dimensionality and spatial distributions using *a* ***single mode-agnostic*** representation

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- ALTO-based tensor decomposition operations proved to ***outperform*** an oracle that selects the best state-of-the-art format due to
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- ALTO achieves **84% parallel efficiency** in high reuse cases, 63% for all cases
  - **7.9x** (*13.2x w/ rank specialization*) **geometric-mean speedup** over the best general formats
  - **4.3x geometric-mean compression ratio** over the best mode-specific formats

Future Work

- Working on distributed-memory execution of key tensor decomposition algorithms

- Exploring other massively parallel platforms (GPGPUs)

- Investigating additional sparse tensor kernels/algorithms
  ➔ provide an **ALTO library**

ALTO pre-print: **arxiv.org/abs/2102.10245**

Backup

Tensor	Dimensions	#NNZs	Density	Fib. reuse
LBNL	$1.6K \times 4.2K \times 1.6K \times 4.2K \times 868.1K$	$1.7M$	4.2×10^{-14}	Limited
NIPS	$2.5K \times 2.9K \times 14K \times 17$	$3.1M$	1.8×10^{-06}	High
UBER	$183 \times 24 \times 1.1K \times 1.7K$	$3.3M$	3.8×10^{-04}	High
CHICAGO	$6.2K \times 24 \times 77 \times 32$	$5.3M$	1.5×10^{-02}	High
VAST	$165.4K \times 11.4K \times 2 \times 100 \times 89$	$26M$	7.8×10^{-07}	High
DARPA	$22.5K \times 22.5K \times 23.8M$	$28.4M$	2.4×10^{-09}	Limited
ENRON	$6K \times 5.7K \times 244.3K \times 1.2K$	$54.2M$	5.5×10^{-09}	High
NELL-2	$12.1K \times 9.2K \times 28.8K$	$76.9M$	2.4×10^{-05}	High
FB-M	$23.3M \times 23.3M \times 166$	$99.6M$	1.1×10^{-09}	Limited
FLICKR	$319.7K \times 28.2M \times 1.6M \times 731$	$112.9M$	1.1×10^{-14}	Limited
DELI	$532.9K \times 17.3M \times 2.5M \times 1.4K$	$140.1M$	4.3×10^{-15}	Medium
NELL-1	$2.9M \times 2.1M \times 25.5M$	$143.6M$	9.1×10^{-13}	Medium
AMAZON	$4.8M \times 1.8M \times 1.8M$	$1.7B$	1.1×10^{-10}	High
PATENTS	$46 \times 239.2K \times 239.2K$	$3.6B$	1.4×10^{-03}	High
REDDIT	$8.2M \times 177K \times 8.1M$	$4.7B$	4.0×10^{-10}	High